A witty title containing ‘homotopy algebras’, ‘scattering amplitudes’, ‘holography’ and ‘Schwinger–Keldysh’, such as this one

Sprita titolo inkluzivanta «homotopiajn alĝebrojn», «disĵetajn amplitudojn», «holografion» kaj «Schwinger-Keldiŝ», ekz. ĉi tiu mem

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Summary *Resumo*

The homological perturbation lemma for L_∞-algebras naturally capture perturbation theories. Applied to physics, it captures

- Feynman diagrams for S-matrix
- Witten diagrams for AdS/CFT
- Cut diagrams for Schwinger–Keldysh observables

etc. These involve natural extensions of the basic HPL.

La homologia perturba lemo por L_∞-alĝebroj nature priskribas perturbajn teoriojn. Aplikate al fiziko, ĝi priskribas

- *Diagramoj de Feynman por la S-matrico*
- *Diagramoj de Witten por AdS/KKT*
- *Trancĉitaj diagramoj por observeblaĵoj de Schwinger-Keldiŝ*

ktp. Tiuj postulas naturajn ĝeneraligojn de la baza HPL.

Chain complex *Ĉenkomplekso*

$$\mathfrak{g} = (\cdots \xrightarrow{d} \mathfrak{g}_0 \xrightarrow{d} \mathfrak{g}_1 \xrightarrow{d} \cdots)$$

$$d^2 = 0$$

boundary of boundary vanishes

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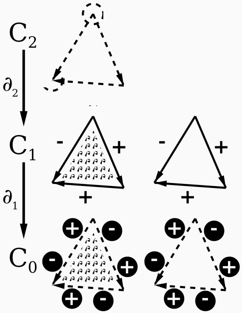


Differential graded Lie algebra

Diferenciala grada alĝebro de Lie

$$\mathfrak{g} = (\cdots \xrightarrow{d} \mathfrak{g}_0 \xrightarrow{d} \mathfrak{g}_1 \xrightarrow{d} \cdots)$$

$$[\ ,\ ]\colon \mathfrak{g}_i \otimes \mathfrak{g}_j \to \mathfrak{g}_k$$

$$\begin{aligned} \mathrm{d}(\mathrm{d}x) &= 0 \\ \mathrm{d}[x,y] - [\mathrm{d}x,y] - [x,\mathrm{d}y] &= 0 \\ [x,[y,z]] + [y,[z,x]] + [z,[x,y]] &= 0 \end{aligned}$$

L_∞-algebra *L_∞-alĝebro*

$$d = \mu_1 \colon \mathfrak{g} \to \mathfrak{g}$$

$$[\ ,\ ] = \mu_2 \colon \mathfrak{g} \otimes \mathfrak{g} \to \mathfrak{g}$$

$$\vdots$$

$$\mu_i \colon \underbrace{\mathfrak{g} \otimes \cdots \otimes \mathfrak{g}}_{i} \to \mathfrak{g}$$

$$\sum \mu_i(\mu_j(x_1, \dots, x_i), x_{i+1}, \dots, x_{i+j-1}) = 0$$

e.g. *ekz.*

$$[[x, y], z] + [[y, z], x] + [[z, x], y] - d\mu_3(x, y, z) - \mu_3(dx, y, z) - \mu_3(x, dy, z) - \mu_3(x, y, dz) = 0$$

L_∞-algebra *L_∞-alĝebro*

$$\mathrm{d} = \mu_1 : \mathfrak{g} \to \mathfrak{g}$$

$$[\ ,\ ] = \mu_2 : \mathfrak{g} \otimes \mathfrak{g} \to \mathfrak{g}$$

$$\vdots$$

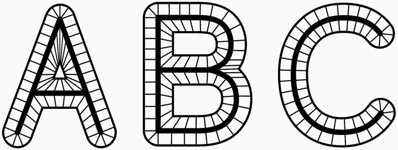
$$\mu_i : \underbrace{\mathfrak{g} \otimes \cdots \otimes \mathfrak{g}}_{i} \to \mathfrak{g}$$

$$\sum \mu_i(\mu_j(x_1, \dots, x_i), x_{i+1}, \dots, x_{i+j-1}) = 0$$

Compatible with metric: *Kongrua kun metriko:*

$$\langle\ ,\ \rangle : \mathfrak{g} \otimes \mathfrak{g} \to \mathbb{R}$$

Deformation retract *Deformoretiro*



$$h \circlearrowright \mathbf{A} \underset{i}{\overset{p}{\rightleftarrows}} \mathsf{A}$$

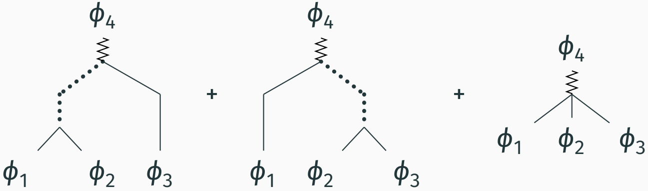
$$p \circ i = \mathrm{id}_{\mathsf{A}}$$

$$i \circ p = \mathrm{id}_{\mathbf{A}} - d \circ h - h \circ d \simeq \mathrm{id}_{\mathbf{A}}$$

Minimal model — *Minimuma modelo*

$$h \circlearrowright \mathfrak{g} \underset{i}{\overset{p}{\rightleftarrows}} \mathrm{H}(\mathfrak{g})$$

$$\downarrow$$



$$S = \int_{\mathbb{R}^d} \frac{1}{2!}\phi D\phi + \frac{1}{3!} f_{abc}\phi^a\phi^b\phi^c + \cdots$$
$$= \langle \phi, \mu_1(\phi)\rangle + \frac{1}{3!}\langle \phi, \mu_2(\phi,\phi)\rangle + \cdots$$

$$\mathfrak{g} = \mathrm{Span}\{\overset{\text{field}}{\phi}, \underset{\text{antif.}}{\phi^+}\}$$
$$\mu_1(\phi)_a = (D\phi)_a$$
$$\mu_2(\phi)_a = f_{abc}\phi^b\phi^c$$
$$\langle \phi, \phi^+\rangle = \int_{\mathbb{R}^d} \phi\phi^+$$

$$\mathfrak{g} = \mathrm{Span}\{\overset{\text{field}}{\phi}, \underset{\text{antif.}}{\phi^+}\}$$

$$\mu_1(\phi)_a = (D\phi)_a$$

$$\mu_2(\phi)_a = f_{abc}\phi^b\phi^c$$

$$\langle \phi, \phi^+ \rangle = \int_{\mathbb{R}^d} \phi\phi^+$$

$$\circlearrowleft \ \mathfrak{g} \rightleftarrows \mathrm{H}(\mathfrak{g})$$

$$\mathrm{H}(\mathfrak{g}) = \mathrm{Span}\{\overset{\text{on-shell f.}}{\ker D}, \underset{\text{on-shell antif.}}{\operatorname{coker} D}\}$$

🌲$_n = \langle \phi_1, \mu_{n-1}(\phi_2, \ldots, \phi_n) \rangle \qquad (n \geq 3)$

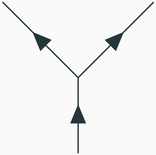
Field theory *Kampa teorio*

$$🌲_n = \langle \phi_1, \mu_{n-1}(\phi_2, \dots, \phi_n) \rangle \qquad (n \geq 3)$$

🤨

➰❓

Δ_{BV} =



Field theory *Kampa teorio*

$$🌲_n = \langle \phi_1, \mu_{n-1}(\phi_2, \ldots, \phi_n) \rangle \qquad (n \geq 3)$$

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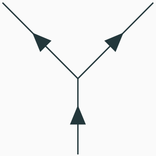
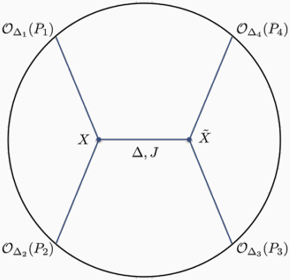
$\Delta_{BV} =$ 

2️⃣⁉️

$$ds^2_{\mathrm{AdS}_{d+1}} = z^{-2}\left(\mathrm{d}z^2 + \sum_{i=1}^{d}(\mathrm{d}y^i)^2\right)$$

$$Z_{\mathrm{AdS_{d+1}}}\left(\phi_i \xrightarrow{z\to 0} \phi_i^\circ\right) = \left\langle \exp(\phi_i^\circ O^i)\right\rangle_{\mathrm{CFT}_d}$$



$$S = \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g} \left(\frac{1}{2} \left((\partial\phi)^2 - m^2 \phi^2 \right) + \frac{1}{3!} \lambda \phi^3 \right)$$

$$= \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g} \left(\frac{1}{2} \phi (\Delta - m^2) \phi + \frac{1}{3!} \lambda \phi^3 \right) + \int_{\mathbb{R}^d = \partial\, \mathrm{AdS}_{d+1}} \phi^\circ N \phi^\circ$$

$$S = \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\left(\frac{1}{2}\left((\partial\phi)^2 - m^2\phi^2\right) + \frac{1}{3!}\lambda\phi^3\right)$$
$$= \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\left(\frac{1}{2}\phi(\Delta - m^2)\phi + \frac{1}{3!}\lambda\phi^3\right) + \int_{\mathbb{R}^d=\partial\,\mathrm{AdS}_{d+1}} \phi^\circ N \phi^\circ$$

$$\mathfrak{g} = \mathrm{Span}\{\phi, \phi^+\}$$
$$\mu_1(\phi) = (\Delta - m^2)\phi$$
$$\mu_2(\phi, \phi') = \lambda\phi\phi'$$
$$\langle\phi, \phi^+\rangle = \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\,\phi\phi$$

$$S = \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\left(\frac{1}{2}\left((\partial\phi)^2 - m^2\phi^2\right) + \frac{1}{3!}\lambda\phi^3\right)$$
$$= \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\left(\frac{1}{2}\phi(\Delta - m^2)\phi + \frac{1}{3!}\lambda\phi^3\right) + \int_{\mathbb{R}^d = \partial\,\mathrm{AdS}_{d+1}} \phi^\circ N \phi^\circ$$

$$\mathfrak{g} = \mathrm{Span}\{\phi, \phi^+\}$$
$$\mu_1(\phi) = (\Delta - m^2)\phi$$
$$\mu_2(\phi, \phi') = \lambda\phi\phi'$$
$$\langle\phi, \phi^+\rangle = \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\,\phi\phi$$

$$\mathrm{H}(\mathfrak{g}) = \mathrm{Span}\{\ker\Delta, \operatorname{coker}\Delta\} = \mathscr{C}^\infty(\mathbb{R}^d) = \mathrm{CFT}_d \text{ source}$$

minimal model = connected Witten diagram

= connected CFT_d correlators

$$\mathrm{H}(\mathfrak{g}) = \mathrm{Span}\{\ker\Delta, \operatorname{coker}\Delta\} = \mathscr{C}^\infty(\mathbb{R}^d) = \mathrm{CFT}_d \text{ source}$$

$$\begin{aligned}\text{minimal model} &= \text{connected Witten diagram}\\ &= \text{connected CFT}_d \text{ correlators}\end{aligned}$$

2-pt function = boundary term

$$\begin{aligned}S &= \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{3!}\lambda\phi^3\right)\\ &= \int_{\mathrm{AdS}_{d+1}} \sqrt{\det g}\left(\frac{1}{2}\phi\Delta\phi + \frac{1}{3!}\lambda\phi^3\right) + \int_{\mathbb{R}^d=\partial\,\mathrm{AdS}_{d+1}} \phi^\circ N \phi^\circ\end{aligned}$$

AdS$_{d+1}$:

$$\int \phi^\circ N \phi^\circ \sim \int_{\mathbb{R}^d} \mathrm{d}y \int_{\mathbb{R}^d} \mathrm{d}y' \frac{\phi^\circ(y)\phi^\circ(y')}{\|y - y\|^\Delta}, \quad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}$$

$$\langle O(y) O(y') \rangle = \frac{1}{\|y - y'\|^\Delta}$$

Minkowski $\mathbb{R}^{d+1}$:

$$\int \phi^\circ N \phi^\circ \sim \int \mathrm{d}^d \vec{p}\, E_{\vec{p}} \phi(\vec{p}) \phi(-\vec{p})$$

$$S[\phi] = \frac{1}{2} \langle \phi, \mu_1(\phi) \rangle + \frac{1}{3!} \langle \phi, \mu_2(\phi, \phi) \rangle + \cdots + \langle \pi_1(\phi), \pi_1(\phi) \rangle_\partial$$

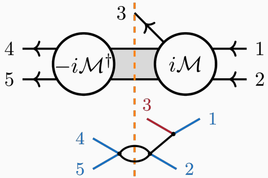
What if we use 🤪 deformation retract?

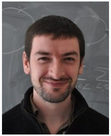
Kio, se oni uzus 🤪an deformoretiron?

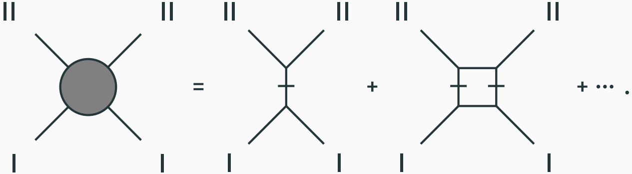
$$(\mathrm{H}(\mathfrak{g}), \tilde{\mu}_i) \rightleftarrows \mathfrak{g} \rightleftarrows (\mathrm{H}(\mathfrak{g}), \mu_i)$$

$$\langle \phi_1, \tilde{\mu}_{n-1}(\phi_2, \dots, \phi_n) \rangle \neq \langle \phi_1, \mu_{n-1}(\phi_2, \dots, \phi_n) \rangle$$

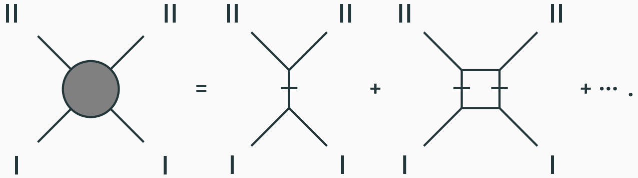
Schwinger–Keldysh *Schwinger-Keldiŝ*







Schwinger–Keldysh *Schwinger-Keldiŝ*



$$\mathfrak{G} = \mathfrak{g}^{(1)} \oplus \mathfrak{g}^{(2)} \oplus \cdots \oplus \mathfrak{g}^{(N)}$$

$$(\tilde{i}, \tilde{p}, \tilde{h}) = (i, p, h + \delta(p^2 + m^2)\Theta(p^0)\sigma_{i+1\leftarrow i} + \delta(p^2 + m^2)\Theta(-p^0)\sigma_{i\leftarrow i+1})$$