Unravelling the holomorphic twist

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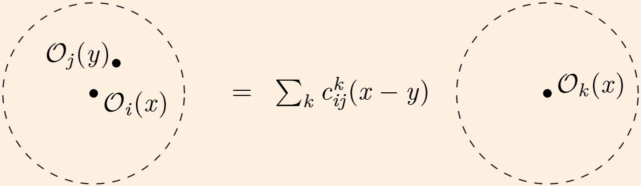
Motivation

OPEs are hard

Correlation functions of local operators are fundamental observables in any QFT

The OPE (together with the vevs) encodes any correlation function of local operators and is strongly constrained by associativity (a.k.a. crossing symmetry)



In general it is incredibly hard to access this data and without conformal symmetry the OPE is typically not convergent

To make progress it is convenient to add symmetries to the problem

In this case: Conformal and/or supersymmetry

Protected sectors in SUSY QFTs

This is most evident in 2D, where the conformal group is enhanced to the infinite-dimensional Virasoro symmetry

In this case the associativity of the OPE, together with these symmetries sometimes allows one to completely solve the theory

Belavin,Polyakov,Zamolodchikov

In higher dimensions, the (super)conformal algebra is finite dimensional so in general we do not expect the same powerful methods to apply

However, in the presence of supersymmetry we can often still obtain exact OPE data in particular protected sectors of the theory

These put strong constraint on the full OPE and are able to provide powerful non-perturbative results

Often these sectors can be obtained by twisting the original theory, or equivalently by passing to the cohomology of a particular nilpotent supercharge

Examples:

- Any 4D $\mathcal{N} = 2$ SCFT contains a sector isomorphic to a 2D VOA
  Beem,Lemos,Liendo,Peelaers,Rastelli,van Rees
- Any 3D $\mathcal{N} = 4$ SCFT contains a sector isomorphic to a 1D topological QM [Pufu,Dedushenko,Yacoby,Fan,Beem,Peelaers,Rastelli]

More generally any observable that is computable through supersymmetric localisation belongs to some protected sector

THE HOLOMORPHIC TWIST

In this talk we consider 4D $\mathcal{N} = 1$ SQFTs

The supercharge to find a 2D VOA subsector is no longer available

Twisting with a generic supercharge results in a 4D holomorphic theory or higher-VOA!

This is the largest possible protected sector and therefore contains much more information than the VOA twist, including black hole states!

The price we have to pay is that this structure is mathematically more complicated and not as well understood as their 2D counterparts

It only has a perturbative definition

In this talk we will mostly consider SCFTs but many of the constructions go through without conformal symmetry where the holomorphic twist provides a powerful RG invariant!

Outline

Warm up: 2D VOAs

Higher-VOAs from holomorphic FTs

The holomorphic twist

Summary and outlook

Warm up: 2D VOAs

The algebraic structure underlying a 2D (chiral) CFT is given by a vertex operator algebra or VOA

- State space V
- State-operator map

$$V \to (\mathrm{End}\, V)[[z]] \;:\; \mathbf{O} \mapsto \mathbf{O}(z) = \sum_{n\in\mathbf{Z}} \frac{\{\mathbf{O}\,,\,\bullet\}_n}{z^{n+1}}$$

- Translation operator T that acts as $(T\mathbf{O})(z) = \partial_z \mathbf{O}(z)$
- Vacuum vector $|0\rangle$ such that $|0\rangle\,(z) = \mathrm{Id}_V$

The OPE can be rewritten in terms of the brackets $\{\bullet\,,\,\bullet\}_n$ as

$$\mathbf{O}_1(z)\mathbf{O}_2(w) = \sum_{n\in\mathbf{Z}} \frac{\{\mathbf{O}_1\,,\,\mathbf{O}_2\}_n\,(w)}{(z-w)^{n+1}}$$

Conversely the brackets are useful to unpack the OPE

$$\{\mathbf{O}_1\,,\,\mathbf{O}_2\}_n(0) = \oint \frac{\mathrm{d}z}{2\pi i} z^n\,\mathbf{O}_1(z)\mathbf{O}_2(0)$$

It can be useful to collect the non-negative modes in a generating function called the λ-bracket

$$\{\mathbf{O}_1\,{}_\lambda\,\mathbf{O}_2\} \equiv \oint_{S^1} \frac{\mathrm{d}z}{2\pi i}\mathrm{e}^{\lambda z}\,\mathbf{O}_1(z)\mathbf{O}_2(0) = \sum_{n\geq 0}\frac{\lambda^n}{n!}\{\mathbf{O}_1\,,\,\mathbf{O}_2\}_n$$

A vector space V equipped with a λ-bracket and a translation operator defines a Lie conformal algebra

Any VOA defines a Lie conformal algebra simply by forgetting the regular part of the OPE

A generalisation of this structure will be very useful in higher-dimensional computations

Note that any holomorphic operator $\mathbf{O}(z)$ can be thought of as a current for a global symmetry with conservation law

$$\bar{\partial}\mathbf{O}(z) = 0\,.$$

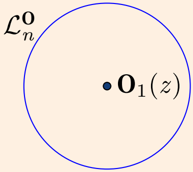
We can then construct a new current $\rho(z)\mathbf{O}(z)$, where ρ is any $\bar{\partial}$-closed function,

$$\rho \in H^{0,0}_{\bar{\partial}}(\mathbf{C}\backslash 0) = \mathbf{C}[z, z^{-1}]$$

so we naturally find an infinite-dimensional symmetry algebra

Phrased differently, the action of the bracket $\{\mathbf{O}, \bullet\}_n$ can be understood as inserting the topological line operator $\mathcal{L}^{\mathbf{O}}_n$ obtained by integrating $z^n\mathbf{O}$ over a circle

$$\{\mathbf{O}, \mathbf{O}_1\}_n(z) =$$



The Virasoro algebra Vir_c is a central extension of the Witt algebra of vector fields on a punctured disc

Choosing a basis $\mathbf{L}_n = -z^{n+1}\partial_z$, the commutation relations are

$$[\mathbf{L}_m, \mathbf{L}_n] = (m-n)\mathbf{L}_{m+n} + c\,\varphi(\mathbf{L}_m, \mathbf{L}_n)\,,$$

the central extension is defined by a $2-$cocycle, $\varphi \in H^2(\mathrm{Witt})$

$$\varphi(\mathbf{L}_m, \mathbf{L}_n) = \frac{1}{12} n(n^2-1)\delta_{m+n,0}$$

In physical applications, the Virasoro algebra is obtained as the mode algebra of the holomorphic stress tensor $T(z)$

$$T(z) = \sum_{n\in\mathbf{Z}} \frac{\mathbf{L}_n}{z^{n+2}}$$

We can define the λ-bracket of the stress tensor with a primary operator as

$$\begin{aligned}\{T\,{}_\lambda\,\mathbf{O}\}(0) &= \oint_{S^1} \frac{\mathrm{d}z}{2\pi i} \mathrm{e}^{\lambda z}\, T(z)\mathbf{O}(0) \\ &= \mathbf{L}_{-1}\mathbf{O}(0) + \lambda\,\mathbf{L}_0\mathbf{O}(0) + \lambda^2\,\mathbf{L}_1\mathbf{O}(0) + \cdots \\ &= \partial\mathbf{O}(0) + h\,\lambda\,\mathbf{O}(0)\end{aligned}$$

From which we can read of the singular OPE

$$T(z)\mathbf{O}(0) \sim \frac{h\,\mathbf{O}(0)}{z^2} + \frac{\partial\mathbf{O}(0)}{z}$$

Similarly for two stress tensors we have

$$\{T\,{}_\lambda\,T\}(0) = \lambda^3 \frac{c}{12} + (2\lambda + \partial)\,T(0)$$

and we recover the usual singular OPE

$$T(z)\,T(0) \sim \frac{c/2}{z^4} + \frac{2\,T(0)}{z^2} + \frac{\partial\,T}{z}$$

Higher-VOAs from holomorphic FTs

The definition in higher dimensions closely follow the 2D ones but the theory is much less developed [Costello, Gwilliam, Williams, ...]

Given any $\bar{\partial}$-closed holomorphic operator $\mathbf{O}(z_1, z_2)$ we find an infinite tower of conserved currents $\rho\,\mathbf{O}(z)$ where ρ are elements of the Dolbeault cohomology

$$\rho \in H^{0,i}_{\bar{\partial}}(\mathbf{C}^2 \backslash 0) = \begin{cases} \mathbf{C}[z_1, z_2]\,, & i = 0 \\ \mathbf{C}[\partial_1, \partial_2]\omega_{\mathrm{BM}}\,, & i = 1 \end{cases}$$

For ρ in the degree 0 part this results in a direct generalisation of the positive modes in 2D

$$\{\mathbf{O}_1\,,\,\mathbf{O}_2\}_{n_1,n_2}(0) = \oint_{S^3} \frac{\mathrm{d}^2 z}{(2\pi i)^2} z_1^{n_1} z_2^{n_2} \mathbf{O}_1(z)\mathbf{O}_2(0) \qquad n_i \geq 0$$

while the negative modes can be obtained by picking ρ from the degree 1 part

$$\{\mathbf{O}_1\,,\,\mathbf{O}_2\}_{n_1,n_2}(0) = \oint_{S^3} \frac{\mathrm{d}^2 z}{(2\pi i)^2} \partial_1^{-n_1-1} \partial_2^{-n_2-1} \omega_{\mathrm{BM}}(z)\mathbf{O}_1(z)\mathbf{O}_2(0) \qquad n_i \leq -1$$

As in 2D these brackets give us ∞ many codim-1 topological defects

Unlike in 2D, these brackets only satisfy the Jacobi identity homotopically and form an L_∞ structure

Higher homotopies are captured by a collection of higher brackets $[\bullet, \cdots, \bullet]_n$ satisfying a collection of Jacobi-like identities

$$\begin{aligned}&[[x_1, x_2]_2\,, x_3]_2 + [[x_2, x_3]_2\,, x_1]_2 + [[x_3, x_1]_2\,, x_2]_2 = \\ &\quad = \mathrm{d}\,[x_1, x_2, x_3]_3 + [\mathrm{d}x_1, x_2, x_3]_3 + [x_1, \mathrm{d}x_2, x_3]_3 + [x_1, x_2, \mathrm{d}x_3]_3\end{aligned}$$

Collecting the non-negative modes we can again define the λ-bracket as follows

$$\{\mathbf{O}_1\,{}_\lambda\,\mathbf{O}_2\} = \oint_{S^3} \frac{\mathrm{d}^2 z}{(2\pi i)^2} \mathrm{e}^{\lambda\cdot z}\mathbf{O}_1(z)\mathbf{O}_2(0)$$

Similarly, for the higher brackets we can introduce higher $n-$ary $\lambda-$brackets

$$\left\{\mathbf{O}_1\,{}_{\lambda_1}\mathbf{O}_2\,{}_{\lambda_2}\,\cdots\,{}_{\lambda_{n-1}}\mathbf{O}_n\right\}$$

which can similarly be defined as integrals over the configuration space of $n-1$ points but this definition quickly becomes very tedious to apply

Example: Higher-Virasoro VOA

The higher-Virasoro algebra 2-Vir$_{A,C}$ is a central extension of the higher-Witt algebra 2-Witt of vector fields on a punctured 2-disc

Analogous to before we define the basis

$$\mathbf{L}^{+}_{m,n} = \rho_{m+1,n}\partial_{+} \qquad \mathbf{L}^{-}_{m,n} = \rho_{m,n+1}\partial_{-}$$

Note that only non-zero when m, n are both non-negative or both negative

The non-vanishing commutators are given by

$$\begin{aligned}
\left[\mathbf{L}^{+}_{m,n}\,,\,\mathbf{L}^{+}_{m',n'}\right] &= (m-m')\,\mathbf{L}^{+}_{m+m',n+n'} \\
\left[\mathbf{L}^{-}_{m,n}\,,\,\mathbf{L}^{-}_{m',n'}\right] &= (n-n')\,\mathbf{L}^{-}_{m+m'+1,n+n'-1} \\
\left[\mathbf{L}^{+}_{m,n}\,,\,\mathbf{L}^{-}_{m',n'}\right] &= n\,\mathbf{L}^{+}_{m+m'+1,n+n'-1} - (m'+1)\,\mathbf{L}^{-}_{m+m',n+n'} \\
\left[\mathbf{L}^{-}_{m,n}\,,\,\mathbf{L}^{+}_{m',n'}\right] &= (m+1)\,\mathbf{L}_{m+m',n+n'} - n'\,\mathbf{L}^{+}_{m+m'+1,n+n'-1}
\end{aligned}$$

Example: Virasoro VOA

The central extensions of the higher-Witt algebra are classified by Gelfand-Fuks cohomology classes $\varphi \in H^2(\text{2-Witt})$ which is known to be two-dimensional [Saberi,Williams]

Unlike in 2D the central extensions do not appear in the 2-bracket but in the 3-bracket

$$\left[\mathbf{L}^{\alpha}_{m,n}\,,\,\mathbf{L}^{\beta}_{k,l}\,,\,\mathbf{L}^{\gamma}_{r,s}\right] = \delta_{m+k+r,0}\delta_{n+l+s,0}\left(A\,\psi_1^{\alpha\beta\gamma} + C\,\psi_2^{\alpha\beta\gamma}\right)$$

As we will see next, these coefficients A and C can be related to the conformal anomalies of 4d $\mathcal{N}=1$ SCFTs

The holomorphic twist

Consider a 4D $\mathcal{N} = 1$ SCFT in Euclidean space

The superconformal algebra is given by $\mathfrak{su}(4|1)$ with generators

$$\mathcal{P}_{\alpha\dot{\alpha}} \quad \mathcal{K}^{\dot{\alpha}\alpha} \quad \mathcal{M}_{\alpha\beta} \quad \mathcal{M}_{\dot{\alpha}\dot{\beta}} \quad \mathcal{Q}_{\alpha} \quad \widetilde{\mathcal{Q}}_{\dot{\alpha}} \quad \mathcal{S}^{\alpha} \quad \widetilde{\mathcal{S}}^{\dot{\alpha}} \quad \mathcal{R}$$

We label fields by their charges (Δ, j_1, j_2, r) under the Cartan or in short $[j_1, j_2]_{\Delta}^{(r)}$

The superconformal algebra contains two spinor supercharges $\mathcal{Q}_{\alpha}$ and $\widetilde{\mathcal{Q}}_{\dot{\alpha}}$, with commutation relations

$$\left\{\mathcal{Q}_{\alpha}\,,\,\widetilde{\mathcal{Q}}_{\dot{\alpha}}\right\} = i\,\mathcal{P}_{\alpha\dot{\alpha}} \qquad \left\{\widetilde{\mathcal{Q}}_{\dot{\alpha}}\,,\,\widetilde{\mathcal{Q}}_{\dot{\beta}}\right\} = \{\mathcal{Q}_{\alpha}\,,\,\mathcal{Q}_{\beta}\} = 0$$

where $\mathcal{P}_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}}\mathcal{P}_{\mu}$ are the translation generators

We can obtain a holomorphic theory by passing to the cohomology of a nilpotent supercharge $\mathbf{Q} = \mathcal{Q}_{-}$

We can obtain a holomorphic theory by passing to the cohomology of a nilpotent supercharge $\mathbf{Q}=\mathcal{Q}_-$

This choice selects a complex structure such that $\{z^{\dot\alpha},\bar z^{\dot\alpha}\}=\{x^{+\dot\alpha},x^{-\dot\alpha}\}$ are (anti-)holomorphic coordinates on $\mathbf{C}^2$

Moreover, notice that

$$\left\{\mathbf{Q},\widetilde{\mathcal{Q}}_{\dot\alpha}\right\}=\partial_{\bar z^{\dot\alpha}}$$

Hence, anti-holomorphic translations are exact and the twisted theory is holomorphic in the cohomological sense

The choice of twisting supercharge breaks the rotation group to SU$(2)_2$ generated by $\mathcal{M}_{\dot\alpha\dot\beta}$

The twisted rotation generator $M_R=M_{+-}-R$ is unbroken and extends the SU(2) to U(2) generating holomorphic rotations of the $z^{\dot\alpha}$

The fields in the twisted theory are labelled by their SU(2)$_1$ Cartan and their R-twisted spin

The space of operators surviving the twist can be defined as the $\mathbf{Q}$-cohomology of local operators

$$\{\mathbf{Q}, \mathcal{O}\} = 0 \qquad \mathcal{O} \neq \{\mathbf{Q}, \mathcal{O}'\} = 0$$

Since the operators are anihilated by one chiral supercharge we call them semi-chiral [Budzik,Gaiotto,Kulp,Williams,Wu,Yu]

In superconformal theories operators can be grouped in multiplets by acting on a superconformal primary with superconformal and special conformal generators [Cordova,Dumitrescu,Intriligator]

$$X\,\bar{Y}[j_L, j_R]^{(r)}_{\Delta} \qquad X,\, Y \in \{L\,, A_1\,, A_2\,, B_1\}$$

For generic Δ supercharges and derivatives act freely and operators in such multiplets are $\mathbf{Q}$-closed only if they are $\mathbf{Q}$-exact

When Δ saturates some BPS bounds some descendent operators are missing and the multiplet becomes short

Harmonic representatives of the cohomology satisfy

$$\{\mathbf{Q}\,,\,\mathbf{Q}^\dagger\} = \Delta + \frac{3}{2}r + \frac{1}{2}j_L = 0$$

Note that any short (chiral) multiplet contributes one operator to the $\mathbf{Q}$-cohomology

The OPE between two semi-chiral operators in necessarily regular hence these operators form the semi-chiral ring

$$\mathcal{O}_1(z)\mathcal{O}_2(0) \sim c_{12}^k \mathcal{O}_k(0) + \mathbf{Q}\text{-exact}$$

Since anti-holomorphic translations are exact, non-holomorphic terms in the OPE are exact

The OPE is at most meromorphic but due to Hartog's theorem there are no functions on $\mathbf{C}^2$ singular at one point hence the OPE is regular

In order to find more interesting structure it will be important to keep track of their descendants

To this end it will be useful to introduce (chiral) superfields

$$\mathbf{O} = \mathrm{e}^{\mathrm{d}\bar{z}^{\dot{\alpha}} \widetilde{\mathcal{Q}}_{\dot{\alpha}}} \mathcal{O} = \mathbf{O}^{(0)} + \mathbf{O}^{(1)} + \mathbf{O}^{(2)}$$

where the usual superspace coordinate $\theta^{\dot{\alpha}}$ transforms as a anti-holomorphic one-form in the twisted theory

Similarly, the supercovariant derivative is given by $D_- = \mathbf{Q} + \bar{\partial}$ and we have the holomorphic descent relations

$$\mathbf{Q}\mathbf{O}^{(k)} + \bar{\partial}\mathbf{O}^{(k-1)} = (\mathbf{Q}\mathbf{O})^{(k)}$$

we call a superfield semi-chiral if

$$D_- \mathbf{O} = 0$$

which simply means that its 0^{th} component is a semi-chiral operator

We can identify the $\mathbf{Q}$-cohomology with the space of semi-chiral superfields modulo the image of D_-

Since the twisted theory is holomorphic we can apply all the tools introduced above

We can introduce the λ-bracket in the twisted theory as

$$\begin{aligned}\{\mathbf{O}_1 \,{}_\lambda\, \mathbf{O}_2\} &= \oint_{S^3} \frac{\mathrm{d}^2 z}{(2\pi i)^2} \mathrm{e}^{\lambda \cdot z} \mathbf{O}_1(z) \mathbf{O}_2(0) \\ &= \int_{\mathbf{C}^2} \frac{\mathrm{d}^2 z}{(2\pi i)^2} \mathrm{e}^{\lambda \cdot z} \mathbf{Q} \left(\mathbf{O}_1(z) \mathbf{O}_2(0) \right)\end{aligned}$$

This integral is only non-vanishing when the integrand is a $(2,2)$ form

Hence we see that this bracket captures the singular terms in the OPEs

$$\mathbf{O}_1^{(1)}(z)\mathbf{O}_1^{(0)}(0) \quad \text{and} \quad \mathbf{O}_1^{(0)}(z)\mathbf{O}_1^{(1)}(0)$$

In perturbation theory we can rewrite this as

[Budzik,Gaiotto,Kulp,Williams,Wu,Yu]

$$\{\mathbf{O}_1\,{}_\lambda\,\mathbf{O}_2\} = \int_{\mathbf{C}^2} \frac{\mathrm{d}^2 z}{(2\pi i)^2} \mathrm{e}^{\lambda\cdot z}\mathbf{Q} :\mathbf{O}_1(z)\mathbf{O}_2(0):$$

where $:\cdots:$ denotes the operation of performing all possible Wick contractions

This definition generalises immediately to higher brackets

$$\{\mathbf{O}_1\,{}_{\lambda_1}\,\cdots\,{}_{\lambda_{n-1}}\,\mathbf{O}_n\} = \prod_{k=1}^{n-1}\int_{\mathbf{C}^2} \frac{\mathrm{d}^2 z_k}{(2\pi i)^2} \mathrm{e}^{\lambda_k\cdot z_k}\mathbf{Q} :\mathbf{O}_1(z_1)\cdots\mathbf{O}_{n-1}(z_{n-1})\mathbf{O}_n(0):$$

For more on the the structure of these brackets and more general brackets encoding also regular the terms in the OPE see Jingxiang's talk next week

In this talk we take a different direction and will look more closely at what aspects of the physical theory are encoded in these brackets

First of all, any $\mathcal{N}=1$ SCFT contains a stress tensor in an $A_1\bar{A}_1[1,1]_3^{(0)}$ multiplet $\mathcal{S}_\mu$ which contains the R-symmetry current as its top component

Therefore this multiplet gives rise to a semi-chiral operator and superfield $\mathbf{S}_{\dot{\alpha}} = D_+\mathcal{S}_{+\dot{\alpha}}$

Similarly, if the theory enjoys a flavour symmetry the spectrum of the theory contains a protected conserved current which sits in a $A_2\bar{A}_2[0,0]_2^{(0)}$ conserved current multiplet $\mathcal{J}$

This multiplet gives rise to a semi-chiral operator and supermultiplet $\mathbf{J} = D_+\mathcal{J}$

In the remainder of this talk we will carefully study the brackets of these operators and show how they encode the anomalies of the physical theory

The binary bracket λ-bracket is completely fixed by SU(2) covariance and anti-symmetry in the adjoint $\mathfrak{g}$ indices

$$\left\{\mathbf{J}^a\,{}_\lambda\,\mathbf{J}^b\right\} = f^{ab}{}_c\mathbf{J}^c$$

and the normalisation fixed the constant factor

Similarly the ternary bracket is fixed by SU(2) covariance, symmetry and the graded commutativity of the bracket to take the form

$$\left\{\mathbf{J}^a\,{}_{\lambda_1}\mathbf{J}^b\,{}_{\lambda_2}\,\mathbf{J}^c\right\} = k\,d^{abc}(\lambda_1, \lambda_2)$$

where d^{abc} is the symmetric invariant tensor on $\mathfrak{g}$

In order to uncover the meaning of k let us explicitly compute the binary and ternary brackets

The OPE between conserved current superfields has been computed for generic interacting field theories [Osborn,Fortin,Intriligator,Stergiou]

Acting with the supercovariant derivatives one can select the correct descendant and explicitly compute the binary bracket to fix the normalisation

The perturbative definition of the ternary bracket does not apply in a generic interactive theory hence we have to modify it a bit

In a free theory the currents are quadratic in free fields so the fully Wick contracted expression is proportional to the identity

Therefore we can compute it equivalently by inserting the three-point function

$$\{\mathbf{J}\,{}_{\lambda_1}\,\mathbf{J}\,{}_{\lambda_2}\,\mathbf{J}\} = \int_{(\mathbf{C}^2)^2} \frac{\mathrm{d}^2 z_1 \mathrm{d}^2 z_2}{(2\pi i)^4} \mathrm{e}^{\lambda_1 \cdot z_1 + \lambda_2 \cdot z_2}\, \bar{\partial}\, \langle \mathbf{J}(z_1)\mathbf{J}(z_2)\mathbf{J}(0)\rangle$$

Computing the relevant descendant three-point function we find that the coefficient k is precisely the G flavour 't Hooft anomaly!

Defining the modes of the current as

$$\mathbf{J}^a_{m,n} \cdot \mathbf{O}(0) \equiv \mathbf{J}^a, \mathbf{O}_{m,n} = \int_{\mathbf{C}^2} \frac{\mathrm{d}^2 z}{(2\pi i)^2} \rho_{m,n} \mathbf{J}^a(z) \mathbf{O}(0)$$

we find the commutation relations

$$\left[\mathbf{J}^a_{m,n}\,,\,\mathbf{J}^b_{k,l}\right] = f^{ab}{}_c \mathbf{J}^c_{m+k,n+l}$$

and the L_∞ three-bracket

$$\left[\mathbf{J}^a_{m,n}\,,\,\mathbf{J}^b_{k,l}\,,\,\mathbf{J}^c_{r,s}\right] = k\, d^{abc} \delta_{m+k+r,0} \delta_{n+k+s,0} (ks - lr)\,.$$

Analogous to the higher-Virasoro VOA this gives a higher analogue of the Kac-Moody VOA

The binary bracket λ-bracket is again completely fixed by SU(2) covariance

$$\left\{\mathbf{S}_{\dot\alpha}\,{}_\lambda\,\mathbf{S}_{\dot\beta}\right\} = \lambda_{\dot\alpha}\mathbf{S}_{\dot\beta} + \lambda_{\dot\beta}\mathbf{S}_{\dot\alpha} + \partial_{\dot\alpha}\mathbf{S}_{\dot\beta}$$

and the normalisation fixed the constant factor

Similarly the ternary bracket is fixed by SU(2) covariance and the graded commutativity of the bracket to take the form

$$\left\{\mathbf{S}_{\dot\alpha}\,{}_{\lambda_1}\mathbf{S}_{\dot\beta}\,{}_{\lambda_2}\,\mathbf{S}_{\dot\gamma}\right\} = A\,\Lambda_{\text{vec}} + C\,\Lambda_{\mathcal{N}=4}$$

where

$$\Lambda_{\text{vector}} = -\frac{1}{24\pi^2}(\lambda_1,\lambda_2)^2\left[\lambda_{1\dot\alpha}\epsilon_{\dot\beta\dot\gamma} + \lambda_{2\dot\beta}\epsilon_{\dot\gamma\dot\alpha} - (\lambda_{1\dot\gamma}+\lambda_{2\dot\gamma})\epsilon_{\dot\alpha\dot\beta}\right]$$
$$\Lambda_{\mathcal{N}=4} = \frac{1}{3\pi^2}\lambda_{1\dot\alpha}\lambda_{2\dot\beta}(\lambda_1+\lambda_2)_{\dot\gamma}(\lambda_1,\lambda_2)\,.$$

Similarly computing the OPE and 3pt function of the appropriate descendant we find that A and C are given by

$$A = \frac{4}{3}(3c-2a) \qquad C = 16(a-c)$$

Summary and outlook

SUMMARY

In talk we have introduced the holomorphic twist of $\mathcal{N} = 1$ SCFTs and shown how it gives rise to a holomorphic field theory

Similar to 2D CFTs, this theory experiences an infinite dimensional symmetry enhancement allowing for many explicit computations

Although the OPEs between semi-chiral operators is regular, we have introduced a variety of higher operations which encode a wealth of information of the physical theory

Two-brackets encode the singular OPE between operators and their descendants, while three-brackets encode anomalies

In particular we have shown how $\mathbf{JJJ}$ brackets encode the 't Hooft anomalies and $\mathbf{SSS}$ brackets encode conformal anomalies
Similarly, one can show that $\mathbf{JJS}$ anomalies encode the flavour levels k_G

Outlook

We have left many questions untouched:

- With extended supersymmetry the twisted theory remains supersymmetric and has a richer structure. One can perform an additional twist and obtain a 2D VOA [Williams,Saberi-WIP]
- So far the holomorphic twist only has a perturbative definition. A non-perturbative description will be instrumental in improving our understanding of black hole states/dualities/...
- In order to improve our understanding it would be useful to have a better understanding of the representation theory of the higher VOAs we encountered [Scheinpflug]
- An alternative approach to obtain non-perturbative insights is by studying the holomorphic twist of Argyres-Douglas theories using their $\mathcal{N} = 1$ Lagrangian description [WIP]
- Many physical operations do not yet have a good understanding in the holomorphic twist, such as Higgsing [WIP]
- Can we extract the full (classical) vacuum moduli space from the holomorphic twist?

Thank you!