Cosmological Graphs from Scattering Amplitudes

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Based on:

2312.13803: Subtle Simplicity of Cosmological Correlators,
2501.10598: Cosmological Dressing Rules
with Arthur Lipstein, Joe Marshall, Jiajie Mei, Ivo Sachs, Pierre Vanhove
+ work in progress

Observables in Flat Space

1. Scattering is one of the simplest process one can study in QFT and QG and allows us to quantify certain observables in flat space.

2. For example: Scattering amplitudes, which are directly related to physical observables (Cross Sections) that are measurable in experiments.

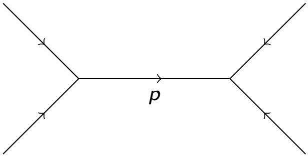
3. To obtain the measurable cross-section we compute "S-matrix^2"

$$\sigma \sim \int d\psi_{\text{out}} \langle \psi_{\text{in}} | S | \psi_{\text{out}} \rangle \langle \psi_{\text{out}} | S | \psi_{\text{in}} \rangle$$

4. Usually observables are computed using in-in correlators, including curved space

1. Typical approach to computing observables – First compute a S-matrix and then integrate it

2. These are usually evaluated via Feynman Diagrams in momentum space



$$= \frac{g^2}{p^2 + m^2}$$

3. These involve off-shell particles and has spurious poles in intermediate steps, involves very heavy algebra.

4. Even in flat space the full computation becomes very hard very soon

1. Very stark in spinning theories. Eg: In Gravity, the 4-point function has 2000+ terms [DeWitt]

2. However many cancellations occur and the final expressions are simple. To quote DeWitt (1967) on the 2→2 graviton scattering computation:

> quanta, eliminate many of the terms from these expressions. Nevertheless, a large amount of cancellation between terms still has to be dug out of the algebra, and this, combined with the fact that the final results are ridiculously simple, leads one to believe that there must be an easier way. The cross sections which one finds are

3. Similar quote in Parke-Taylor in 1986!

4. This "easier way" known as the BCFW recursion was developed in the early 2000's and eventually led to a lot more developments and the "Amplitudes program"

5. Take Home Message: Final answers are often easier than building blocks – Often require special choice of helicities, masses, etc.

Moving Forward

1. How hard is it to compute such in-in correlators in curved space?
2. Can we directly obtain this observable without computing the "amplitude"?
3. One such example: Equal time in-in correlators,

$$\langle \Psi | O(t=0, \vec{x}_1) \cdots O(t=0, \vec{x}_n) | \Psi \rangle$$

   where O's are operators at a time slice. In our universe ($\approx$ FRW) these are cosmological correlators and related to measurements.

4. These often form examples of Schwinger-Keldysh correlators
5. Ground state $|\Psi\rangle$ (wave function of the universe) are related to AdS correlators via analytic continuation.

Conventions

1. Will mainly work in dS_4,

$$ds^2 = \frac{-dt^2 + d\vec{x}^2}{t^2} \implies \sqrt{-g} = \frac{1}{t^4}$$

2. The correlation functions are evaluated at $t = 0$,

$$\langle \Psi | O(t=0, \vec{x}_1) \cdots O(t=0, \vec{x}_n) | \Psi \rangle$$

3. Since there is translational invariance along $\vec{x}$, momentum $\vec{k}$ is defined by

$$O(t=0, \vec{x}) = \int d^3k \; e^{i\vec{k}\cdot\vec{x}} O(t=0, \vec{k})$$

4. "Energy" $= |\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$

Correlator $\sim |\Psi|^2$

1. The in-in correlator is defined as

$$\langle\Psi|\phi(\vec{k}_1)\cdots\phi(\vec{k}_n)|\Psi\rangle = \int D\phi\phi(\vec{k}_1)\cdots\phi(\vec{k}_n)|\Psi(\phi)|^2$$

2. What is the state $\Psi(\phi)$? The most popular choice is the Hartle-Hawking/Bunch-Davies state,

$$\Psi[\varphi(\vec{x})] = \int_{\phi(t=-\infty)=0}^{\phi(t=0,\vec{x})=\varphi(\vec{x})} D\phi\ e^{iS[\phi]}$$

   – By analytical continuation, these are equivalent to computing correlators in AdS [Maldacena]

3. The wave functions $\Psi(\phi)$ are computed using path integrals and perturbation theory is expressed in terms of Witten diagrams.

4. These correlators can also be computed via Schwinger-Keldysh formalism [Weinberg]
   – Also related to Shadow prescription [Sleight, Tarrona] [Di Pietro, Komatsu, Gorbenko]

Wave Function

1. A lot of attention has been given to compute the wave function as it is the building block
2. Lots of good intuition from AdS/CFT as it is related to the AdS correlators [Maldacena, Pimentel; McFadden, Skenderis; Ghosh, Kundu, Raju, Trivedi]
3. Good motivations to do so as even in flat space we usually compute the $\langle out|in\rangle$ correlator first: S-matrix
4. Standard intuition suggests that the building blocks are simpler than final object
5. We will first review Ψ as its conceptually simpler and show some examples where the correlator is nicer and a direct connection with amplitudes

Harmonic Oscillator: Gaussian Wave Function

1. We defined the ground state wave function via path integral

$$\Psi(\phi) = \int_{\varphi(-\infty)=0}^{\varphi(0)=\phi} D\varphi e^{iS}$$

However they also satisfy Schrodinger Equation

$$H\Psi = 0$$

2. For example: Ground state wave function for Harmonic Oscillator

$$H = \frac{d^2}{dx^2} + \omega^2 x^2 \implies \psi(x) = e^{-\frac{1}{2}\omega^2 x^2}$$

3. Similarly for a free scalar field in flat space you integrate over all oscillator modes

$$H = \int d^3k \frac{\partial^2}{\partial\varphi_{\vec{k}}\, \partial\varphi_{-\vec{k}}} + \omega_k^2 \varphi_{\vec{k}}\varphi_{-\vec{k}} \implies \Psi[\varphi] = e^{-\frac{1}{2}\int d^3k\ \omega_k^2 \varphi_{\vec{k}}\varphi_{-\vec{k}}}$$

Would obtain the same result from $e^{-S_{on-shell}}$.

1. Higher order corrections are evaluated perturbatively:
   Either by $(H_{free} + H_{int})\,|\psi\rangle = 0$ [Hatfield] or by solving the path integral

2. Generic structure is of the form:

$$\Psi[\varphi] \sim \exp\Big[\int d^3k_1 d^3k_2\ \Psi_2(\vec{k}_1, \vec{k}_2)\ \varphi(\vec{k}_1)\varphi(\vec{k}_2) + \int d^3k_1 \cdots d^3k_4\ \Psi_4(\vec{k}_1, \cdots, \vec{k}_4)\ \varphi(\vec{k}_1)\cdots\varphi(\vec{k}_4) + \cdots\Big]$$

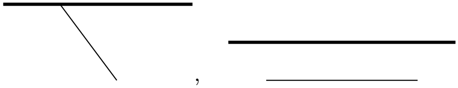
   — $\Psi_n(\vec{k}_1, \cdots, \vec{k}_n)$ are called Wave function coefficients and by analytical continutation, related to AdS correlators.

3. Often known as Old Fashioned Perturbation Theory and is clearly non-covariant

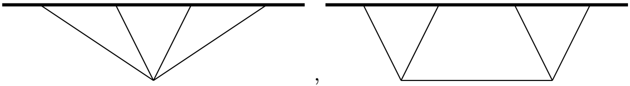
Witten Diagrams

1. Perturbation theory can be expressed in terms of Witten Diagrams

2. No time-translation invariance



   Bulk-Boundary & Bulk-Bulk Propagators.
   Example: for Ψ_4 in ϕ^4 and ϕ^3 theory:



3. Momentum conservation along spatial directions only.

Example of a Witten diagram

1. Consider Conformally Coupled scalar field.
   – Related to massless scalar fields in flat space via Weyl Transformation.

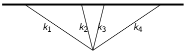
2. Propagators are (Notation: $k \equiv |\vec{k}|$)

$$\phi_c(t; k) = te^{ikt},$$

$$G(t, t'; k) = \frac{tt'}{2k}\Big[\underbrace{\theta(t-t')e^{ik(t-t')} + \theta(t'-t)e^{ik(t'-t)}}_{Feynman} - \underbrace{e^{ik(t+t')}}_{B.C}\Big]$$

Satisfies Dirichlet boundary conditions hence not translational inv.

3. Example: Contribution to tree-level ψ_4 in ϕ^4 theory,

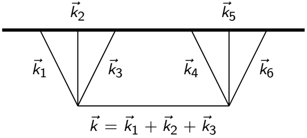


$$= \int_{-\infty}^{0} dt e^{i(k_1+\cdots+k_4)t} \underbrace{\int d^3x e^{i\sum_i \vec{k}_i\cdot\vec{x}}}_{\text{spatial mom cons.}} = \frac{\delta(\vec{k}_1 + \cdots + \vec{k}_4)}{k_1 + k_2 + k_3 + k_4}$$

— Time integrals are cut-off at $t = 0$ (Need correct $i\epsilon$ prescriptions at $\eta \to -\infty$)
— Hence instead of getting $\delta(k_1 + k_2 + k_3 + k_4)$ we get a poles $(k_1 + k_2 + k_3 + k_4)^{-1}$! (related to flat space limits [Raju])

Exchange Diagram: ϕ^4

Consider an exchange diagram in ϕ^4 theory: $\left(k = |\vec{k}_1 + \vec{k}_2 + \vec{k}_3| \text{ and } k_{ijm} = |\vec{k}_i| + |\vec{k}_j| + |\vec{k}_m|\right)$



$$= \int_{-\infty}^{0} dt_1 dt_2 e^{ik_{123}t_1} e^{ik_{456}t_2} G(t_1, t_2, \vec{k})$$

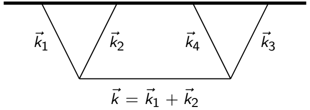
$$= \frac{1}{(k_{123} + k_{456})(k + k_{123})(k + k_{456})}$$

$$= \frac{1}{(k_{123} + k_{456})} \Psi_4(k, k_1, k_2, k_3) \Psi_3(k, k_4, k_5, k_6) \implies \text{Product of Lower Point } \Psi$$

1. No energy conservation
2. These poles have a physical meaning (eg: flat space limit [Raju])
3. Recursive formulas via IBP [Arkani-Hamed, Benincasa, Postnikov] including spinning theories [Albayrak, CC, Kharel; CC, Chowdhury, Moga, Singh]

Exchange Diagram: ϕ^3

Consider an exchange diagram in ϕ^3 theory: $\left(k = |\vec{k}_1 + \vec{k}_2| \text{ and } k_{ij} = |\vec{k}_i| + |\vec{k}_j|\right)$



$$= \int_{-\infty}^{0} \frac{dt_1\, dt_2}{(t_1 t_2)^2} e^{i(k_1+k_2)t_1} e^{i(k_3+k_4)t_2} G(t_1, t_2, \vec{k}_1 + \vec{k}_2)$$

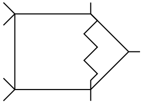
1. Computing the time integrals gives [Arkani-Hamed, Maldacena]

$$\Psi_4 = \frac{1}{1-u-v}\left[\mathrm{Li}_2(1-u) + \mathrm{Li}_2(1-v) + \log(u)\log(v) - \frac{\pi^2}{6}\right]$$

   where $u = \frac{k_{12}+k}{k_{12}+k_{34}}$, $v = \frac{k_{34}+k}{k_{12}+k_{34}}$

2. Exactly equal to 7-pt pentagon! [Drummond, Henn, Trnka] [WIP with S. Prabhu and P. Raman]

$$\Psi_4 = \frac{1}{1-u-v}$$



3. Hence already see dS at tree level $\sim$ Loops in Amplitudes

Correlator

1. Notice that we still need the correlator (upto overall factors)

$$
\begin{aligned}
&\langle\Psi|\phi_1\cdots\phi_4|\Psi\rangle \\
&=\int D\phi\phi_1\cdots\phi_4 e^{-\mathrm{Re}\Psi(\phi)} \\
&=\int D\phi\ \phi_1\cdots\phi_4 e^{-\int \mathrm{Re}\psi_2\phi^2}\Big(1+\int \mathrm{Re}\Psi_3\phi_1\phi_2\phi_3+\int \mathrm{Re}\Psi_4\phi_1\phi_2\phi_3\phi_4 \\
&\qquad\qquad +\frac{1}{2}\int \mathrm{Re}\Psi_3\phi_1\phi_2\phi_3\int \mathrm{Re}\Psi_3\phi_1'\phi_2'\phi_3'+\cdots\Big) \\
&\sim \mathrm{Re}\Psi_4+\frac{\mathrm{Re}\Psi_3\mathrm{Re}\Psi_3}{\mathrm{Re}\Psi_2}
\end{aligned}
$$

2. For higher loops/points expression gets more complicated [Benincasa, Dian] , eg.,

$$
\langle\phi_1\cdots\phi_4\rangle^{(3)}=\psi_4^{(3)}+\frac{\psi_6^{(2)}}{\psi_2}+\frac{\psi_4^{(1)}\psi_4^{(1)}\psi_4^{(1)}}{4\psi_2\psi_2\psi_2\psi_2}+\frac{\psi_4^{(1)}\psi_6^{(1)}}{2\psi_2\psi_2\psi_2}+\frac{\psi_4^{(1)}\psi_4^{(2)}}{2\psi_2\psi_2}+\frac{\psi_8^{(1)}}{\psi_2\psi_2}
$$

3. Claim: the final sum of the RHS is much simpler than inital expectations [CC, Lipstein, Mei, Sachs, Vanhove] [also see recent work by Glew]

Status

1. Scalar trees are often difficult and in general lot of connections left to uncover. [Benincasa, Arkani-Hamed, Baumann, Pimentel, Pajer, Henn, Raman, Jazayeri, Sleight, Taronna · · ·]
2. Only explicit loop integrals done now are mostly axi-symmetric (bubbles, necklace, banana) [CC, Carmi, Chowdhury, Lipstein, Moga, Mei, Sachs, Singh, Vanhove, Benincasa, Brunello, Mandal, Mastrolia, Vazao, Bertan, Heckelbacher, de la Cruz, Skvortsov, · · ·]
3. No MHV type computation beyond 4 points even in Yang-Mills [Raju]
4. Three different expressions of 4-graviton correlator and not obvious how they are related to each other [Raju; Bonifacio, Goodhew, Joyce, Pajer, Stefanyszyn; Armstrong, Goodhew, Lipstein, Mei]
5. Most computations are for the wave functions but we eventually need in-in correlators.
6. In the next few slides I will describe a new representation for the in-in correlator expressed in terms of flat space Feynman diagrams

Wave functions

1. To keep things simple we revisit CC scalars with ϕ^4 interaction and consider a single exchange

2. This is equivalent to a 6-pt function ψ_6,

$$\begin{aligned}\psi_6 &= \int_0^\infty dt_1 dt_2 e^{-k_{123}t_1} e^{-k_{456}t_2} \frac{1}{2k}\left[\Theta_{12}e^{-kt_{12}} + \Theta_{21}e^{-kt_{21}} - e^{-k(t_1+t_2)}\right] \\ &= \int_0^\infty dt_1 dt_2 e^{-k_{123}t_1} e^{-k_{456}t_2} \int_{-\infty}^\infty \frac{dp \sin(pt_1)\sin(pt_2)}{p^2+k^2} \\ &= \int_{-\infty}^\infty \frac{dp p^2}{(p^2+k_{123}^2)(p^2+k_{456}^2)} \frac{1}{P^2}, \qquad P_\mu = (p, \vec{k})\end{aligned}$$

where I simply interchanged order of integration and performed integrals over t_1, t_2.

3. Looks like an energy integral over a flat space amplitude

4. However, does not work in the same way for loops and higher points

5. Still need to compute correlator by adding ψ_n's

Correlator

1. However there exists a similar representation for the in-in correlator too

$$\langle\phi_1\cdots\phi_6\rangle^{(1)} = \psi_6 + \frac{\psi_4\psi_4}{\psi_2} = \frac{1}{(k_{123}+k_{456})(k_{123}+k)(k_{456}+k)} + \frac{1}{k(k_{123}+k)(k_{456}+k)}$$
$$= \int_{-\infty}^{\infty} dp \frac{k_{123}k_{456}}{(p^2+k_{123}^2)(p^2+k_{456}^2)}\frac{1}{P^2}, \qquad P_\mu = (p, \vec{k})$$

   Comparing to wave function, the only difference is the factor in numerator

2. However, for the in-in correlator this Kernel works at arbitrary points and loops.

3. Example: at one loop we have, [CC, Lipstein, Mei, Sachs, Vanhove]

$$\langle\phi_1\cdots\phi_4\rangle^{(2)} = \int_{-\infty}^{\infty} dp \frac{k_{12}k_{34}}{(p^2+k_{12}^2)(p^2+k_{34}^2)} \int \frac{d^4L}{L^2(L+P)^2}$$

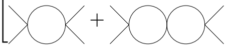
4. The Kernel is generally theory dependent

5. In contrast to a Celestial Amplitude, this integral is on a single Feynman diagram (and not full amplitude)

Correlator

1. Hence the following representation is true at 4-pts for a class of diagrams,

$$\langle \phi_1 \cdots \phi_4 \rangle^{(s)} = \int_{-\infty}^{\infty} \frac{dp k_{12} k_{34}}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \left[\quad + \quad + \cdots \right]$$

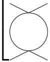


where the Kernel is written for the s-channel.

Note: Energy conservation exists for the flat space amplitude.

2. Different channels/topologies have a different kernel's depending on the external momenta. Example: t-channel's Kernel

$$\langle \phi_1 \cdots \phi_4 \rangle^{(t)} = \int_{-\infty}^{\infty} \frac{dp k_{41} k_{23}}{(p^2 + k_{41}^2)(p^2 + k_{23}^2)} \left[\quad + \cdots \right]$$



3. Natural generalization to higher points
Example: Triangle Diagram,

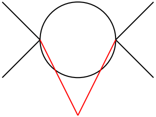
$$\langle \phi_1 \cdots \phi_6 \rangle^{(s)} = \int_{-\infty}^{\infty} \frac{dp_1 dp_2 k_{12} k_{34} k_{56}}{(p_1^2 + k_{12}^2)(p_1^2 + k_{34}^2)((p_1 + p_2)^2 + k_{56}^2)} \int \frac{d^4 L}{L^2 (L^2 + P_1)^2 (L + P_2)^2}$$

Not IR/UV divergent.

1. Rule: Take a Feynman graph, integrate over energies upon multiplying with a Kernel – gives a cosmological correlator.
   — will refer to Kernel as Auxiliary Propagators

2. Diagrammatically,

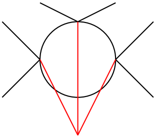
$$\langle \phi_1 \cdots \phi_4 \rangle^{(s)} =$$



$$= \int_{-\infty}^{\infty} \frac{dp\;\; k_{12} k_{34}}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \int \frac{d^4 L}{L^2 (L+P)^2}$$

where $P^\mu = (p, \vec{k})$ & the red lines denote the 1-D propagators, carrying energy
Example: Triangle

$$\langle \phi_1 \cdots \phi_6 \rangle^{(s)} =$$



$$= \int_{-\infty}^{\infty} \frac{dp_1 dp_2 k_{12} k_{34} k_{56}}{(p_1^2 + k_{12}^2)(p_1^2 + k_{34}^2)((p_1 + p_2)^2 + k_{56}^2)} \int \frac{d^4 L}{L^2 (L^2 + P_1)^2 (L + P_2)^2}$$

Higher Masses

1. There are a concrete set of dressing rules worked out for the CC and massless theories with polynomial interations [CC, A. Lipstein, J. Marshall, J. Mei, I. Sachs]

2. CC scalar: ϕ^4 interaction [closely tied to in-out formalism [Donath, Pajer]]

$$\text{Auxiliary Propagators}: \quad \Delta(k_{ext}, p) = \frac{k_{ext}}{k_{ext}^2 + p^2}$$

3. CC scalar: ϕ^3 interaction,

$$\Delta^{(1)}_{k,p} = \int_0^\infty ds \Delta(p, k+s), \qquad \Delta^{(2)}_{k,p} = \pi$$

4. Example, for tree level massless scalars, [to appear with A. Lipstein, J. Marshall, J. Zhang]

$$\Delta_{massless}(k_1, k_2, p) = \partial_{k_1}\partial_{k_2} \int_0^\infty \frac{ds^2}{k_1 k_2}(1 - p\partial_p)\Delta(k_{12} + s, p)$$

— Gives the IR regularized correlator

5. Almost similar dressing formulas work for spinning theories [to appear ''

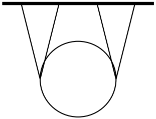
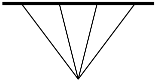
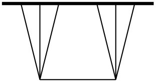
Loops: Simplicity

1. The dressing rule also leads to simplification at loop level

2. Normally one can compute in-in loops by first computing wave function trees/loops and then squaring (or using SK). For example,

$$\langle \phi_1 \cdots \phi_4 \rangle^{(2)} = \Psi_4^{(2)} + \frac{\Psi_4^{(1)}\Psi_4^{(1)}}{2\Psi_2^{(0)}\Psi_2^{(0)}} + \frac{\Psi_6^{(1)}}{2\Psi_2^{(0)}}$$

where

$\Psi_4^{(2)} =$  , $\Psi_4^{(1)} =$  , $\Psi_6^{(1)} =$ 

3. Individually these compute AdS corelators. In particular, consider $\Psi_4^{(2)}$, [CC, Albayrak, Kharel] [Salcedo, Lee, Melville, Pajer]

$$\Psi_4^{(2)} = \frac{\pi}{k_{12}+k_{34}}\frac{1}{\epsilon} + \log^2\left(\frac{k_{34}-k}{k+k_{12}}\right) + \log^2\left(\frac{k_{12}-k}{k+k_{34}}\right) + 2\text{Li}_2\frac{k+k_{34}}{k-k_{12}} + 2\text{Li}_2\frac{k+k_{12}}{k-k_{34}} + \log(\cdots)$$

Loops: Simplicity & Transcendentality

1. Example: However the final correlator is a log [Lee; CC, Lipstein, Mei, Sachs, Vanhove]

$$\langle\phi_1\cdots\phi_4\rangle = \int_{-\infty}^{\infty} dp\,\frac{k_{12}k_{34}}{(p^2+k_{12}^2)(p^2+k_{34}^2)}\int\frac{d^4L}{L^2(L+K)^2}$$
$$\sim \int_{-\infty}^{\infty} dp\,\frac{k_{12}k_{34}}{(p^2+k_{12}^2)(p^2+k_{34}^2)}\log\left(\frac{p^2+k^2}{\Lambda^2}\right)$$
$$= \frac{\pi}{k_{12}+k_{34}}\left[\log\left(\frac{(k_{12}+k)(k_{34}+k)}{\Lambda^2}\right)+\frac{k_{12}+k_{34}}{k_{12}-k_{34}}\ln\left(\frac{k_{34}+k}{k_{12}+k}\right)\right]$$

2. Preserving conformal invariance requires a careful regularization [Senatore, Zaldarriaga] (also see recent work by [Jain, Pajer, Tong])

$$\langle\phi_1\cdots\phi_4\rangle = \frac{\pi}{k_{12}+k_{34}}\left[\log\left(\frac{(k_{12}+k)(k_{34}+k)H^2}{\Lambda^2(k_{12}+k_{34})^2}\right)+\frac{k_{12}+k_{34}}{k_{12}-k_{34}}\ln\left(\frac{k_{34}+k}{k_{12}+k}\right)\right]$$

   – The new branch $\log(k_{12}+k_{34})^2$ also predicted by loop intrgral space limit [Raju]

3. Example shows how the transcendentality drops.

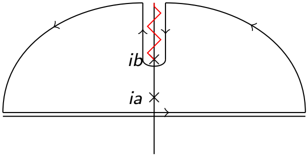
4. While the dressing rule is not needed, it makes it easier to see why it happens.

Loops: Bubble

1. From the dressing rule we get an integral of kind,

$$\int_{-\infty}^{\infty} \frac{dp}{p^2 + a^2} \log(p^2 + b^2) \sim \pi \log$$

In the complex p plane,



$$: \int_{-\infty}^{\infty} dp = \operatorname*{Res}_{p=ia} + \int_{disc}, \quad \int_{disc} = \int_{ib}^{\infty} \frac{dp}{p^2 + a^2}\pi$$

2. Same conclusion for any Li_n insertion. For this class of integrals, we can easily write this as a Polylog recursion by partial fractions

3. However things start getting more complicated when we encounter $\sqrt{p^2 + c^2}$,

Examples: $\displaystyle\int_{-\infty}^{\infty} \frac{dp}{(p^2 + a^2)\sqrt{p^2 + c^2}} \log(p^2 + b^2) \sim \mathrm{Li}_2, \quad \text{Triangle/Box} \sim \frac{\mathrm{Li}_2(\cdots)}{\sqrt{\lambda(p_1, p_2, p_3)}}$

4. Discontinuity across a sq-root branch cut is still sq-root

Loops: Triangle

1. For special 1-loop polygons simplicity is explained by the pole structure of the $\int d^3l$ integrand [CC, Chowdhury, Moga, Singh] [Benincasa, Brunello, Mandal, Mastrolia, Vazao]

$$\int d^4L\ f(l_0,\vec{l}) = \int d^3l \int_{-\infty}^{\infty} dL_0 f(L_0,\vec{l}) = \int d^3l f(\vec{l})$$

2. Since $\langle\cdots\rangle$ comes from Ψ they share many poles but differ in one [Lee]

3. At 1-loop: $\langle\cdots\rangle$ has $\frac{1}{l}$ (is similar to an S-matrix pole), whereas Ψ has $\frac{1}{E_{tot}+l}$

4. Example:
For tadpole,

$$\Psi : \int \frac{d^3l}{l+2k} \sim k^2 \log k \qquad \text{vs} \qquad \langle\cdots\rangle : \int \frac{d^3l}{l} \sim l^2$$

For the bubble:

$$\Psi : \int \frac{d^3l}{(k_{12}+l+|\vec{l}+\vec{k}|)(k_{34}+l+|\vec{l}+\vec{k}|)(k_{12}+k_{34}+l)} \sim \mathrm{Li}_2$$

$$\text{vs} \qquad \langle\cdots\rangle : \int \frac{d^3l}{(k_{12}+l+|\vec{l}+\vec{k}|)(k_{34}+l+|\vec{l}+\vec{k}|)l} \sim \log$$

Loops: Triangle

1. Non-trivial example: Triangle diagram: no analytical expression exists yet

2. The wave function coefficients contains this pole,

$$\int d^3 l \frac{1}{(k_{12}+k_{34}+k_{56}+2l)(k_{12}+l+l')(k_{34}+l+l'')(k_{56}+l'+l')(k_{12}+k_{34}+l+l')}$$

   The master integrals were argued to be Elliptic using differential equations

   [Benincasa, Brunello, Mandal, Mastrolia, Vazao]

3. Combing previous argument with this, correlator is expected to be Polylogarithmic, [CC, Lipstein, Marshall, Mei, Sachs]

$$\int d^3 l \frac{1}{(l)(k_{12}+l+l')(k_{34}+l+l'')(k_{56}+l'+l')(k_{12}+k_{34}+l+l')}$$

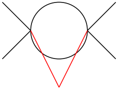
   Consistent with dressing rule as square-roots contain quadratic polynomials

4. However this argument does not fix order of the Polylog

5. Can hope to employ a strategy similar to flat space, where you consider cuts of a diagram [Abreu, Britto, Duhr, Gardi]

Cuts and Discs.

1. Correlator = Flat space diagram + Extra 1D propagators,

$$B(k_{12}, k_{34}) =$$  $$= \int_{-\infty}^{\infty} \frac{dp \;\, k_{12} k_{34}}{(p^2 + k_{12}^2)(p^2 + k_{34}^2)} \int \frac{d^4 L}{L^2 (L+P)^2}$$

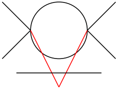
Hence you can consider cuts and set things on-shell [to appear CC, S. Jazayeri, A. Lipstein, J. Marshall, J. Mei, I. Sachs]

2. Can relate to disc. [Cutkosky; t'Hooft, Veltman] , sequential disc., etc. [Bourjaily, Hannesdottir, McLeod, Schwartz, Vergu] [Benincasa, McLeod, Vergu]

3. Cuts of auxiliary propagators are relatively simple.
   – Similar to mass cuts in amplitudes

4. Cutting all auxiliary propagtors recovers the flat space limit (including energy conserving delta function).

Ex:

 $$= \int_{-\infty}^{\infty} dp \delta(p^2 + k_{12}^2)\delta(p^2 + k_{34}^2) \int \frac{d^4 L}{L^2 (L+P)^2} = \delta(k_{12} - k_{34}) \int \frac{d^4 L}{L^2 (L+P)^2}$$

Sum Rules

1. Further, away from the plane $k_{12} = k_{34}$, we get new constraints via relating the cus with discontinuities

$$\mathrm{disc}_{x^2} f(x^2) = f(x^2 + i\epsilon) - f(x^2 - i\epsilon)$$

2. Predicts new Sum Rules for the correlator via sequential disc.,

$$B(k_{12}, k_{34}) + B(k_{12}, -k_{34}) + B(-k_{12}, k_{34}) + B(-k_{12}, -k_{34}) = 0$$

   - Extends to higher pt/loop diagrams (more relations via partial energy sum rules).
   - Also noted for contact graphs in [Donath, Pajer]

3. No such rule is applicable to wave functions!
   - Flat Space Limit/Partial Energy Singularities for ψ can't be written as sequential Disc.

Conclusion

1. Obtained an integral representation for in-in correlators in terms of flat space Feynman diagrams for some theories.

2. New connections between Amplitudes and Cosmological Graphs

3. Integral representations gives new constraints for correlator, loops, etc.

Future Directions

1. Sum of channels via BCFW, similar to AdS [Raju] ?
2. Generalization to FRW, connection with kinematic flow? [Baumann, Goodhew, Joyce, Lee, Pimentel, Westerdijk] [Glew, Pokraka] [Capuano, Ferro, Lukowski, Palazio]
3. How sensitive is this story to Bunch-Davies, eg: any similar structure for correlators in $\alpha-$vaccua? [WIP with Shibam Das, Nilay Kundu]
4. What can we learn about UV properites of correlators and wave functions/BPHZ in dS? [Creminelli, Renaux-Petel, Tambalo, Yingcharoenrat] [Herderschee] [WIP with Suvrat Raju]
5. Does this structure sustain after Renormalization, both UV and IR [Bzowski, McFadden, Skenderis] ?
6. How to handle higher masses [Raman, Yang] [Liu, Qin, Xianyu] ? Can one exploit the tools developed for the wave function [Benincasa] ?
7. Lessons from previous cutting rules/tree theorems [Benincasa; Melville, Pajer; Salcedo, Melville] ?
8. Do similar dressing formulas exist for OTOC like asymptotic observables in flat space [Caron-Huot, Giroux, Hannesdottir, Mizera] ?
9. Other correlators $\langle\psi|\phi\pi\cdots|\psi\rangle$ and unequal time correlators [Kitching, Heavens] ?
10. Resumming graphs [Starobinsky; Senatore, Gorbenko; Cespedes, Davis, Wang] ?