A New Action for Superstring Field Theory

CH 2508.03902

String Field Theory (SFT)

Particle $x^\mu(\tau)$

String $x^\mu(\tau, \sigma)$

Field $\phi(x^\mu)$

String Field Ψ

String spectrum: Infinite set of particles

Ψ encodes infinite set of fields

$\Psi[\phi(x), h_{\mu\nu}(x), \ldots, D_{\mu\nu\rho\sigma}(x), \ldots]$

SFT Action, symmetries

Action + symmetries for component fields

String Field Theory (SFT)

- SFT Feynman diagrams Gives string amplitudes
- Allows systematic treatment of IR divergences, unitarity, UV finiteness. Mass renormalisation
- Non-perturbative formulation: solutions, tachyon condensation ….
- NS5 branes and KK monopoles are solitons
- D-branes from open string boundaries
- D-instanton calculations

Superstring Field Theory?

- No way of constructing kinetic term for Ramond sector

- IIB string has massless RR 4-form C with self-dual field-strength

$$G = dC \qquad\qquad G = *G$$

A IIB superstring field theory would give an action for C, but constructing such an action is a long-standing problem

Sen's Superstring Field Theory

2 string fields! $\Psi, \tilde{\Psi}$

Allows Ramond-sector kinetic term

Two strings:

1) The physical interacting superstring, including graviton

2) A 2nd free string that decouples from physical fields

This doesn't couple to gravity!

Sen's Action for Self-Dual Gauge Fields

- Action for 2 self-dual gauge fields:

1) Physical gauge field C interacting with gravity and other physical fields

2) 2nd gauge field $\tilde{C}$ decoupling from gravity, C, and all physical fields.

Couples only to Minkowski metric

3) Diffeomorphism-like symmetry under which physical gauge field and metric

transform, but 2nd gauge field doesn't transform!

Issues with Sen's S-D Gauge Fields

- Can't put Minkowski metric on most spacetimes, highly restrictive
- ALL fields must change under changes of coordinates and so ALL must transform under diffeomorphisms
- But all fields should couple to gravity? Equivalence principle etc?
- 2nd gauge field $\tilde{C}$ has negative "energy" so just as well it doesn't couple to gravity

New Action for S-D Gauge Fields

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- Replace Minkowski metric with 2nd metric $\tilde{g}_{\mu\nu}$
- Physical sector: $C, g_{\mu\nu}, \ldots$ includes all physical fields
- Shadow sector $\tilde{C}, \tilde{g}_{\mu\nu}$ decouples from physical sector
- TWO diffeomorphism-like symmetries, one for each sector
- Real diffeomorphisms diagonal subgroup, acting on ALL fields
- BI-METRIC Geometry: Novel couplings, non-linear in both metrics

Sen's super-SFT

- Gives correct quantum calculations for physical superstring
- Gives Sen's action for SD gauge field + shadow

Mamade+Zwiebach

- "Exotic Diffeomorphisms": only fields from Ψ transform, but some have strange transformations. Fields from $\tilde{\Psi}$ invariant.
- Action depends on a choice of background — restricts applicability?

New super-SFT

CH 2508.03902

- Reduces to Sen's in certain limit
- Gives correct quantum calculations for physical superstring
- 2 superstrings, one physical and one shadow, but they decouple from each other. Both fully self-interacting.
- Background independent
- 2 "exotic diffeomorphism" symmetries, one for each sector
- Real diffeomorphisms from diagonal subgroup

Free String Field Theory

1st idea: Function on loop space $\Psi[x(\sigma)]$

Siegel: State in 1st-quantised string theory $\Psi \in \mathscr{H}$

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle$$

BRST charge Q

Field equation: $Q\Psi = 0$

Gauge symmetry $\delta\Psi = Q\Lambda$

Solutions modulo gauge symmetry: BRST cohomology classes!

CFT for SFT

CFT with c=26

Ghosts b,c

BRST charge Q

State in Hilbert space

$|\Psi\rangle = \Psi|0\rangle$

Corresponds to operator Ψ

Basis of vertex operators $V_I(x)$

$$\Psi = \sum_I \int dx\, \phi^I(x) V_I(x)$$

Spacetime Fields $\phi^I(x)$

Field equation:

$Q\Psi = 0$

Gives field equations for $\phi^I(x)$

Gauge symmetry

$\delta\Psi = Q\Lambda$

Gives gauge symmetry for $\phi^I(x)$

CFT for SFT

String field Ψ Grassmann even, ghost number two

$L_0^- \Psi = 0, \qquad b_0^- \Psi = 0 \qquad\qquad L_0^- = L_0 - \bar{L}_0, \qquad b_0^- = b_0 - \bar{b}_0$

$$S = \frac{1}{2} \langle \Psi, Q\Psi \rangle$$

SFT Inner product has insertion of $\quad c_0^- = c_0 - \bar{c}_0$

$$\langle A, B \rangle = \langle A \,|\, c_0^- \,|\, B \rangle_{BPZ}$$

CFT for SFT

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SFT Inner product has insertion of $\quad c_0^- = c_0 - \bar{c}_0$

$$\langle A, B \rangle = \langle A \,|\, c_0^- \,|\, B \rangle_{BPZ}$$

E.g. for graviton in
D=26 Minkowski space

$$\Psi \sim \int \frac{d^D p}{(2\pi)^D}\, h_{\mu\nu}(p)\; c\bar{c}\partial X^\mu \bar{\partial} X^\nu e^{ip_\mu X^\mu}$$

Bosonic Closed String Field Theory

Zwiebach

$$S = \frac{1}{2}\langle \Psi, Q\Psi \rangle + \sum_{n=3}^{\infty} \frac{\kappa^{n-2}}{n!}\{\Psi^n\}$$

n-brackets $[A_1, A_2, \ldots, A_n]$ Maps $\mathscr{H}^n \to \mathscr{H}$

$$[A] = QA$$

Satisfy L_∞ algebra

Bosonic Closed String Field Theory

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$$[A] = QA$$

Satisfy L_∞ algebra

$$\{A_1, A_2, \ldots, A_n\} = \langle A_1, [A_2, \ldots, A_n]\rangle$$

$$\{A, A, \ldots, A\} = \{A^n\}$$

Related to correlation function of vertex operators $A_1, A_2, \ldots, A_n$ inserted on S^2

Bosonic Closed String Field Theory

Zwiebach

$$S = \frac{1}{2}\langle \Psi, Q\,\Psi \rangle + \sum_{n=3}^{\infty} \frac{\kappa^{n-2}}{n\,!}\{\Psi^n\}$$

Field equation $\mathscr{F} = 0$

$$\mathscr{F} \;=\; Q\Psi + \frac{\kappa}{2}[\Psi,\Psi] + \frac{\kappa^2}{3\,!}[\Psi,\Psi,\Psi] + \ldots = \frac{1}{\kappa}[e^{\kappa\Psi}]$$

$[\Psi] = Q\Psi$

Gauge symmetry

$$\delta\,|\,\Psi\rangle = \; Q\,|\,\Lambda\rangle + \sum_{n=1}^{\infty} \frac{\kappa^{n-1}}{n\,!}\,[\Lambda\Psi^n]$$

Gravity from SFT

D=26 Minkowski space

Graviton
$$\Psi \sim \int \frac{d^D p}{(2\pi)^D}\, h_{\mu\nu}(p)\; c\bar{c}\partial X^\mu \bar{\partial} X^\nu e^{ip_\mu X^\mu}$$

$$S_{grav} = \frac{1}{2}\int d^{26}x \left(h^{\mu\nu}\partial^2 h_{\mu\nu} + 2(\partial^\nu h_{\mu\nu})^2 \right) + O(\kappa)$$

CH+Zwiebach

Gravity gauge symmetry from SFT gauge symmetry

Sen's Type II Superstring FT

c=15 SCFT + b,c ghosts + β, γ superghosts

or bosonised superghosts η, ξ, ϕ

$\mathscr{H}_{p,q}$ states of left-moving picture number p, right-moving picture number q

String fields, $\Psi, \tilde{\Psi}$ Grassmann even, ghost number 2, GSO even

$$\Psi \in \mathscr{H}_c \equiv \mathscr{H}_{-1,-1} \oplus \mathscr{H}_{-1,-\frac{1}{2}} \oplus \mathscr{H}_{-\frac{1}{2},-1} \oplus \mathscr{H}_{-\frac{1}{2},-\frac{1}{2}},$$

$$\widetilde{\Psi} \in \widetilde{\mathscr{H}}_c \equiv \mathscr{H}_{-1,-1} \oplus \mathscr{H}_{-1,-\frac{3}{2}} \oplus \mathscr{H}_{-\frac{3}{2},-1} \oplus \mathscr{H}_{-\frac{3}{2},-\frac{3}{2}}$$

NS-NS $\oplus$ NS-R $\oplus$ R-NS $\oplus$ R-R

In kernel of $L_0^- = L_0 - \bar{L}_0, \quad b_0^- = b_0 - \bar{b}_0$

Sen’s Type II Superstring FT

$$S = -\frac{1}{2}\langle\widetilde{\Psi}, Q\,\mathscr{G}\,\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}, Q\,\Psi\rangle + \sum_{n=3}^{\infty}\frac{\kappa^{n-2}}{n!}\{\Psi^n\}$$

$$\mathscr{G} : \tilde{\mathscr{H}}_c \to \mathscr{H}_c$$

$\mathscr{G} = 1$ on NS-NS sector

$$\mathcal{G} = \begin{cases} 1 & \text{on } \mathcal{H}_{-1,-1} \\ \chi_0 & \text{on } \mathcal{H}_{-1,-3/2} \\ \bar{\chi}_0 & \text{on } \mathcal{H}_{-3/2,-1} \\ \chi_0\bar{\chi}_0 & \text{on } \mathcal{H}_{-3/2,-3/2} \end{cases}$$

Picture changing operator χ

Sen's Type II Superstring FT

$$S = -\frac{1}{2}\langle\widetilde{\Psi}, Q\,\mathscr{G}\,\widetilde{\Psi}\rangle + \langle\widetilde{\Psi}, Q\,\Psi\rangle + \sum_{n=3}^{\infty}\frac{\kappa^{n-2}}{n!}\{\Psi^n\}$$

Gauge symmetry

$$\delta|\widetilde{\Psi}\rangle = Q|\widetilde{\Lambda}\rangle + \sum_{n=1}^{\infty}\frac{\kappa^{n-1}}{n!}[\Lambda\Psi^n]$$

$$\delta|\Psi\rangle = Q|\Lambda\rangle + \sum_{n=1}^{\infty}\frac{\kappa^{n-1}}{n!}\mathscr{G}[\Lambda\Psi^n]$$

Field Equations

$$Q\Psi + \mathscr{G}\sum_{n=2}^{\infty}\frac{\kappa^{n-1}}{n!}[\Psi^n] = 0$$

In NS-NS sector $\mathscr{G} = 1$: same form as for bosonic string

Defining $$\hat{\Psi} = \Psi - \mathscr{G}\widetilde{\Psi}$$

$$Q\hat{\Psi} = 0$$

Free string

$$\delta|\hat{\Psi}\rangle = Q|\hat{\Lambda}\rangle \qquad \hat{\Lambda} = \Lambda - \mathscr{G}\widetilde{\Lambda}$$

Graviton Action

Strings in D=10 Minkowski space: SCFT X^μ, ψ^μ

$$\Psi_{grav} = \int \frac{d^Dp}{(2\pi)^D} \left(\frac{1}{2} h_{\mu\nu}(p)\ c\bar{c}\,\psi^\mu \tilde{\psi}^\nu e^{-\phi} e^{-\bar{\phi}} \right) e^{ip\cdot X} \qquad \widetilde{\Psi}_{grav} = \int \frac{d^Dp}{(2\pi)^D} \left(\frac{1}{2} \tilde{h}_{\mu\nu}(p)\ c\bar{c}\,\psi^\mu \tilde{\psi}^\nu e^{-\phi} e^{-\bar{\phi}} \right) e^{ip\cdot X}$$

$$S_{grav} = \frac{1}{2} \int d^{10}x \left(h^{\mu\nu} \partial^2 h_{\mu\nu} + 2(\partial^\nu h_{\mu\nu})^2 \right) - \frac{1}{2} \int d^{10}x \left(\hat{h}^{\mu\nu} \partial^2 \hat{h}_{\mu\nu} + 2(\partial^\nu \hat{h}_{\mu\nu})^2 \right)$$

$$\hat{h}_{\mu\nu} = h_{\mu\nu} - \tilde{h}_{\mu\nu}$$

$$\delta h_{\mu\nu} = \partial_\mu \lambda_\nu + \partial_\nu \lambda_\mu \qquad \delta \hat{h}_{\mu\nu} = \partial_\mu \hat{\lambda}_\nu + \partial_\nu \hat{\lambda}_\mu$$

Strings in a Background

For strings moving in a spacetime M with metric $\bar{g}_{\mu\nu}$

Free SCFT $\rightarrow$ Non-linear sigma model with target M

Free graviton propagating on M

$$S_{grav} = \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\left(h^{\mu\nu}\bar{\nabla}^2 h_{\mu\nu} + 2(\bar{\nabla}^\nu h_{\mu\nu})^2\right)$$

Background dependent

Background Independent Action

$$S_{EH}(g) = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}R(g)$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\delta g_{\mu\nu} = \mathscr{L}_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

Diffeomorphism symmetry

$$\delta h_{\mu\nu} = \alpha_{\mu\nu}, \qquad \delta \bar{g}_{\mu\nu} = -\kappa\alpha_{\mu\nu}$$

Shift symmetry

$$\delta h_{\mu\nu} = \nabla_\mu \lambda_\nu + \nabla_\nu \lambda_\mu, \qquad \delta \bar{g}_{\mu\nu} = 0$$

Not an independent symmetry:
Follows from diffeos and shifts

Free SFT Gravity in Background

$$S_{grav} = \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\left(h^{\mu\nu}\,\bar{\nabla}^2 h_{\mu\nu} + 2(\bar{\nabla}^{\nu}h_{\mu\nu})^2\right) - \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\left(\hat{h}^{\mu\nu}\,\bar{\nabla}^2 \hat{h}_{\mu\nu} + 2(\bar{\nabla}^{\nu}\hat{h}_{\mu\nu})^2\right)$$

Diffeomorphism symmetry inherited from Sigma Model

$$\delta\bar{g}_{\mu\nu} = \mathscr{L}_{\xi}\bar{g}_{\mu\nu} = \bar{\nabla}_{\mu}\xi_{\nu} + \bar{\nabla}_{\nu}\xi_{\mu}, \qquad \delta h_{\mu\nu} = \mathscr{L}_{\xi}h_{\mu\nu}, \qquad \delta\hat{h}_{\mu\nu} = \mathscr{L}_{\xi}\hat{h}_{\mu\nu}$$

Free SFT Gravity in Background

$$S_{grav} = \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\,\left(h^{\mu\nu}\,\bar{\nabla}^2 h_{\mu\nu} + 2(\bar{\nabla}^\nu h_{\mu\nu})^2\right) - \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\,\left(\hat{h}^{\mu\nu}\,\bar{\nabla}^2 \hat{h}_{\mu\nu} + 2(\bar{\nabla}^\nu \hat{h}_{\mu\nu})^2\right)$$

Diffeomorphism symmetry inherited from Sigma Model

$$\delta\bar{g}_{\mu\nu} = \mathscr{L}_\xi \bar{g}_{\mu\nu} = \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu, \qquad \delta h_{\mu\nu} = \mathscr{L}_\xi h_{\mu\nu}, \qquad \delta\hat{h}_{\mu\nu} = \mathscr{L}_\xi \hat{h}_{\mu\nu}$$

2 further spin-2 gauge symmetries if $\bar{g}$ Einstein:

$$\delta h_{\mu\nu} = \bar{\nabla}_\mu \lambda_\nu + \bar{\nabla}_\nu \lambda_\mu \qquad \delta\hat{h}_{\mu\nu} = \bar{\nabla}_\mu \hat{\lambda}_\nu + \bar{\nabla}_\nu \hat{\lambda}_\mu$$

Background dependence means these are distinct from diffeomorphisms

Full Sen SFT Gravity

Sen SFT has non-polynomial interactions for Ψ, quadratic in $\tilde{\Psi}$

Interactions for h, not for $\bar{h}$

$$S = S_{EH}(\bar{g} + \kappa h) - \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\left(\hat{h}^{\mu\nu}\,\bar{\nabla}^2\hat{h}_{\mu\nu} + 2(\bar{\nabla}^\nu\hat{h}_{\mu\nu})^2\right)$$

$$S_{EH}(g) = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}R(g)$$

Not background independent

Symmetries of Sen SFT Gravity

$$S = S_{EH}(\bar{g} + \kappa h) - \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\left(\hat{h}^{\mu\nu}\bar{\nabla}^2\hat{h}_{\mu\nu} + 2(\bar{\nabla}^\nu\hat{h}_{\mu\nu})^2\right)$$

$$\delta\bar{g}_{\mu\nu} = \mathscr{L}_\xi\bar{g}_{\mu\nu}, \qquad \delta h_{\mu\nu} = \mathscr{L}_\xi h_{\mu\nu}, \qquad \delta\hat{h}_{\mu\nu} = \mathscr{L}_\xi\hat{h}_{\mu\nu}$$

$$\delta g_{\mu\nu} = \mathscr{L}_\xi g_{\mu\nu} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu$$

Diffeomorphism symmetry

$$\delta h_{\mu\nu} = \nabla_\mu\lambda_\nu + \nabla_\nu\lambda_\mu, \qquad \delta\bar{g}_{\mu\nu} = 0, \qquad \delta\hat{h}_{\mu\nu} = 0$$

"Exotic Diffeomorphisms"

$$\delta h_{\mu\nu} = 0 \qquad \delta\hat{h}_{\mu\nu} = \bar{\nabla}_\mu\hat{\lambda}_\nu + \bar{\nabla}_\nu\hat{\lambda}_\mu, \qquad \delta\bar{g}_{\mu\nu} = 0$$

SFT symmetries give "exotic diffeos" mixed in with trivial symmetries plus field-dependent corrections…

Mamade+Zwiebach

RR Sector and Self-Dual Forms

IIB string has massless RR 4-form C with self-dual field-strength

$$G = dC \qquad\qquad G = *\,G$$

In D=10 Minkowski space background:

Ψ gives sd 5-form $F = *\,F$ $\qquad$ $\tilde{\Psi}$ gives 4-form $\tilde{A}$

$$S = \int -\frac{1}{2} d\tilde{A} \wedge {*d\tilde{A}} + F \wedge d\tilde{A} + O(\kappa)$$

*: Hodge dual wrt Minkowski metric

Agrees with Sen's action for self-dual gauge fields $\qquad$ Mamade+Zwiebach

$$S = \int -\frac{1}{2} d\tilde{A} \wedge {*d\tilde{A}} + F \wedge d\tilde{A}$$

Shift $$F' = F - \frac{1}{2}(d\tilde{A} + {*d\tilde{A}})$$

$$S = \int F' \wedge d\tilde{A}$$

Looks like topological BF theory

But $F' = * F'$

Field equations: $$dF' = 0 \implies F' = dA'$$

$$d\tilde{A} = * d\tilde{A} \implies \tilde{F} = * \tilde{F}$$

Two sd gauge fields!

CH+Lambert

Interesting d=10-CFT, higher dimensional analogue of $\beta\gamma$ system

Coupling to Gravity

F couples to graviton h, $\tilde{F}$ does not

F transforms under “exotic diffeos”, $\tilde{F}$ does not

$$G = F + M(h, F)$$

Full field equations: $dG = 0$

$G = *_g G$ Self-dual wrt metric $g = \eta + \kappa h$

Coupling to Gravity

$G = *_g G$ Self-dual wrt metric $g = \eta + \kappa h$

Replace background η with $\bar{g}$:

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$\tilde{F} = *_{\bar{g}} \tilde{F}$ Sd wrt $\bar{g}$

$dG = 0, \quad d\tilde{F} = 0$

<u>Physical sector:</u> g, G Coupling to each other and other physical fields

<u>Shadow sector:</u> $\tilde{F}, \bar{g}$ Coupling only to each other,
NOT to physical fields

$G = F + M(g, \bar{g}, F)$ Novel bi-metric geometric structure

Democratic Action for All RR

$$S = \int F' \wedge d\tilde{A}$$

$$F' = F'_1 + F'_3 + F'_5 + F'_7 + F'_9$$

$$\tilde{A} = \tilde{A}_0 + \tilde{A}_2 + \tilde{A}_4 + \tilde{A}_6 + \tilde{A}_8 + \tilde{A}_{10}$$

$$F' = *\,F' \qquad\qquad F'_r = *\,F'_{10-r}$$

Interaction between Ψ and $\tilde{\Psi}$ crucial

But final field strengths $G, \tilde{F}$ do not interact

Extending Sen SFT Gravity

Sen SFT has non-polynomial interactions for Ψ, quadratic in $\tilde{\Psi}$

Interactions for h, not for $\bar{h}$

$$S = S_{EH}(\bar{g} + \kappa h) \; - \; \frac{1}{2}\int d^{10}x\sqrt{-\bar{g}}\left(\hat{h}^{\mu\nu}\,\bar{\nabla}^2\hat{h}_{\mu\nu} + 2(\bar{\nabla}^{\nu}\hat{h}_{\mu\nu})^2\right)$$

$$S_{EH}(g) = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}R(g)$$

Fully background independent generalisation:

$$S = S_{EH}(\bar{g} + \kappa h) \; - S_{EH}(\bar{g} + \hat{\kappa}\hat{h})$$

Interactions for both h and $\bar{h}$

New Super SFT Action

$$S = -\tfrac{1}{2}\langle \widetilde{\Psi}, Q\,\mathscr{G}\,\widetilde{\Psi}\rangle + \langle \widetilde{\Psi}, Q\,\Psi\rangle + \sum_{n=3}^{\infty}\frac{\kappa^{n-2}}{n!}\{\Psi^n\} \; - \sum_{n=3}^{\infty}\frac{\hat{\kappa}^{n-2}}{n!}\{\hat{\Psi}^n\}$$

$$\hat{\Psi} = \Psi - \mathscr{G}\widetilde{\Psi}$$

Field Equations

$$Q\hat{\Psi} + \mathscr{G}\sum_{n=2}^{\infty}\frac{\hat{\kappa}^{n-1}}{n!}[\hat{\Psi}^n] = 0 \qquad\qquad Q\Psi + \mathscr{G}\sum_{n=2}^{\infty}\frac{\kappa^{n-1}}{n!}[\Psi^n] = 0$$

Two copies of interacting superstring

Physical string Ψ decoupled from shadow string $\tilde{\Psi}$

Gauge Symmetries

$$\delta|\widetilde{\Psi}\rangle = \; Q|\widetilde{\Lambda}\rangle + \sum_{n=1}^{\infty} \frac{\kappa^{n-1}}{n!} [\Lambda\Psi^n] - \sum_{n=1}^{\infty} \frac{\hat{\kappa}^{n-1}}{n!} [(\Lambda - \mathscr{G}\widetilde{\Lambda})\hat{\Psi}^n]$$

$$\delta|\Psi\rangle = \; Q|\Lambda\rangle + \sum_{n=1}^{\infty} \frac{\kappa^{n-1}}{n!} \mathscr{G} [\Lambda\Psi^n]$$

Two string field parameters $\Lambda, \tilde{\Lambda}$

Gauge Symmetries

$$\delta|\widetilde{\Psi}\rangle = Q|\widetilde{\Lambda}\rangle + \sum_{n=1}^{\infty}\frac{\kappa^{n-1}}{n!}[\Lambda\Psi^n] - \sum_{n=1}^{\infty}\frac{\hat{\kappa}^{n-1}}{n!}[(\Lambda - \mathscr{G}\widetilde{\Lambda})\hat{\Psi}^n]$$

$$\delta|\Psi\rangle = Q|\Lambda\rangle + \sum_{n=1}^{\infty}\frac{\kappa^{n-1}}{n!}\mathscr{G}[\Lambda\Psi^n]$$

Transformation of $\hat{\Psi} = \Psi - \mathscr{G}\widetilde{\Psi}$

$$\delta|\hat{\Psi}\rangle = Q|\hat{\Lambda}\rangle + \mathscr{G}\sum_{n=1}^{\infty}\frac{\hat{\kappa}^{n-1}}{n!}[\hat{\Lambda}\hat{\Psi}^n]$$

$$\hat{\Lambda} = \Lambda - \mathscr{G}\widetilde{\Lambda}$$

Same form as $\delta|\Psi\rangle$

Real diffeomorphisms: in diagonal subgroup $\Lambda = \hat{\Lambda}$

Background Independence

Theory of a graviton h in background $\bar{g}$

Background independent if: $S_{\bar{g}}(h) = S(\bar{g} + \kappa h)$

For bosonic SFT, expect classical solution $\bar{\Psi}$ to correspond to a CFT.

This will have a BRST charge $Q_{\bar{\Psi}}$, brackets $[A, B, \ldots]_{\bar{\Psi}}$
Gives SFT $S_{\bar{\Psi}}(\Psi)$

1. Background independent action if $S_{\bar{\Psi}}(\Psi) = S(\bar{\Psi} + \kappa\Psi)$

2. Check a marginal deformation of CFT gives shift $\bar{\Psi} \to \bar{\Psi} + \delta\bar{\Psi}$

Background Independence

1. Background independent action if $S_{\bar{\Psi}}(\Psi) = S(\bar{\Psi} + \kappa\Psi)$

2. Check a marginal deformation of CFT gives shift $\bar{\Psi} \to \bar{\Psi} + \delta\bar{\Psi}$

2: SFT defined by marginal deformation of CFT and SFT defined by

shifting background related by field redefinition

Shown for Bosonic SFT: Sen and Zwiebach

And for Super SFT: Sen

Background Independence

1. Background independent action if $S_{\bar{\Psi}}(\Psi) = S(\bar{\Psi} + \kappa\Psi)$

2. Check a marginal deformation of CFT gives shift $\bar{\Psi} \to \bar{\Psi} + \delta\bar{\Psi}$

1: True for bosonic SFT

$$Q_{\bar{\Psi}}A = [Ae^{\bar{\Psi}}] = QA + [\bar{\Psi}A] + \frac{1}{2}[\bar{\Psi}^2 A] + \ldots$$

Brackets

$$[A_1, A_2, \ldots, A_n]_{\bar{\Psi}} = [A_1, A_2, \ldots, A_n, e^{\bar{\Psi}}]$$

Background Independence

1. Background independent action if $S_{\bar{\Psi}}(\Psi) = S(\bar{\Psi} + \kappa\Psi)$

2. Check a marginal deformation of CFT gives shift $\bar{\Psi} \to \bar{\Psi} + \delta\bar{\Psi}$

1: <u>False</u> for Sen's superSFT $S(\Psi, \tilde{\Psi})$
(but field equation for Ψ is background independent)

Background Independence

1. Background independent action if $S_{\bar{\Psi}}(\Psi) = S(\bar{\Psi} + \kappa\Psi)$

2. Check a marginal deformation of CFT gives shift $\bar{\Psi} \to \bar{\Psi} + \delta\bar{\Psi}$

1: True for new superSFT. For background $\bar{\Psi}$ satisfying e.o.m.

$$Q\bar{\Psi} + \mathcal{G}\sum_{n=2}^{\infty}\frac{\kappa^{n-1}}{n!}[\bar{\Psi}^n] = 0$$

$$S_{\bar{\Psi}}(\Psi, \tilde{\Psi}) = S(\bar{\Psi} + \kappa\Psi, \tilde{\Psi})$$

(Setting $\kappa = \hat{\kappa}$)

Conclusions

- SFT: two decoupled type II or heterotic superstrings.
- Sen: one interacting, one free. Background dependent, coordinate dependent. “Exotic diffeomorphisms” with some fields invariant.
- New: both interacting. Background independent, coordinate independent.
- Two gravitons, so two spin-2 gauge invariances. Diffeomorphisms diagonal subgroup.
- Key point: The SFT gives correct superstring amplitudes

- Novel bi-metric geometry giving doubled gauge symmetries
- Gives a democratic doubled supergravity action, including self-dual gauge field for IIB
- Field equations decouple, but action involves interactions between 2 sectors, allowing action for self-dual fields and a democratic formulation
- Doubled approach to chiral bosons can be used for quantum heterotic string. 2 world-sheet metrics so 2 modular invariances CH+Lambert

Questions

- Why this doubling? Is it essential? Is there another way?
- Doubling seems to be needed for SFT action based on NSR superstring (in "small formalism").
- Other approaches? Berkovits or GS string? PST treatment of self-dual fields?
- Secret dependence on class of backgrounds. E.g. for string theory on a torus, infinite set of fields $\rightarrow$ infinite set of doubled fields. Gives two decoupled super-DFTs.
- Geometric understanding of SFT interactions?