The magic of scattering amplitudes

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Hertfordshire Mathematical Physics Seminar

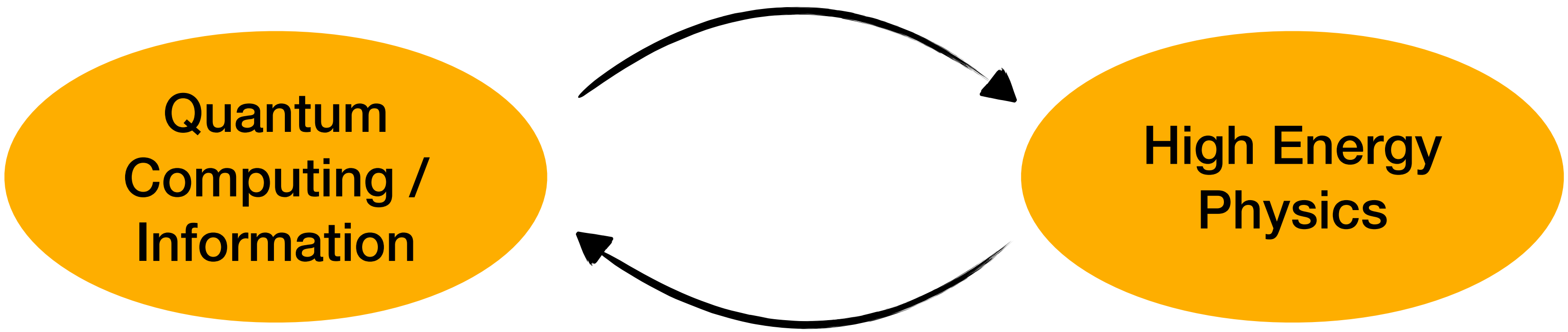
Overview

How is high energy physics relevant for Quantum Information / Computing, and vice versa?

- Quantum entanglement at the LHC.
- Brief introduction to Quantum Computing / Information.
- The property of *magic* of quantum states.
- Applications in collider physics (hep-ph, hep-ex).
- Applications in theoretical physics (hep-th).

Motivation

- In recent years, there has been increasing dialogue between two very large subfields in physics.



- The aim is to use colliders such as the LHC to perform fundamental tests of quantum mechanics…
- …thus complementing table-top tests in **condensed matter** or **optics**.
- Much of this work has focused around **entanglement.**

Entanglement

- Quantum entanglement refers to correlations beyond those possible in classical mechanics.
- A classic example is an entangled state of two spin-1/2 particles:

$$|\psi\rangle = \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

- If we separate the two particles spatially, a measurement of one collapses the spin of the other…
- …regardless of which component is used to measure the spin.
- This **non-local** behaviour has been well-verified in decades of low energy experiments.

Entanglement at the LHC

- The LHC (and similar colliders) excel at producing pairs of spin-1/2 particles…
- …that are then separated in the detector!
- Thus, the possibility arises of performing entanglement measurements.
- There is an excellent recent review of this area (Barr, Fabbrichesi, Floreanini, Gabrielli, Marzola).
- Experiments have reported high-profile results.
- Caution may be needed in their interpretation (Dreiner, Abel).

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Beyond entanglement

- Given the interest in entanglement, we should ask:

What other quantities from Quantum Theory might be interesting for high energy physics?

- Quantum Theory itself has become the inter-related fields of *Quantum Computing* and *Quantum Information Theory*.

- Questions include:

How do we manipulate quantum systems?

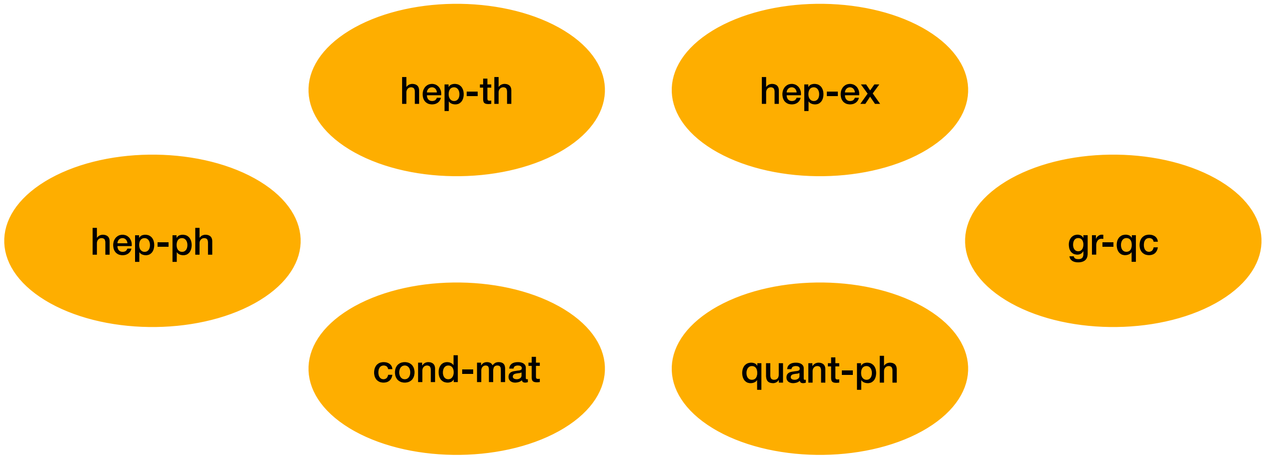
Can we build quantum computers?

What are quantum computers good for?

How do we transmit information in noisy channels?

The language of quantum theory

- Current research in quantum theory unites many areas of the arXiv!



- The languages can often be very different.
- In this talk, we will focus on a quantity called *magic*, which is highly topical in quant-ph research.
- To see why, we need to delve into quantum computing in more detail.

A bit of quantum computing

- In quantum computers, classical bits (with values {0,1}) are replaced by *qubits*:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Orthogonal basis states

where the complex coefficients satisfy $|\alpha|^2 + |\beta|^2 = 1$.

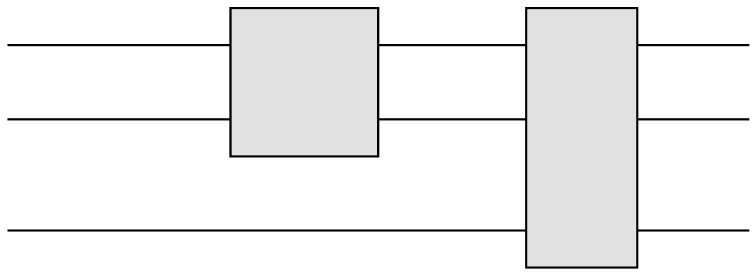
- Example: a spin-1/2 particle is a single "qubit", where the above states are spin states.

- For multi-qubit systems, a choice of basis states is

$$|\psi_1\psi_2\ldots\psi_n\rangle \equiv |\psi_1\rangle \otimes |\psi_2\rangle \otimes \ldots \otimes |\psi_n\rangle$$

Quantum computers

- Quantum computers take qubits, and subject them to unitary transformations.
- We can draw circuit diagrams, with fancy symbols to represent the transformations (“quantum gates”):



- These are the equivalent of logic gates in classical computers…
- …and change the quantum state at each intermediate step.

- The gates have names like *Hadamard*, *phase*, *CNOT*, *Pauli* etc.
- We will not need the precise details.

Stabiliser states

- Quantum computers are expected to vastly outperform classical computers.
- Naïvely, this is due to quantum *superposition* and *entanglement*.
- However, this not quite true.
- To see why, we need the concept of a *stabiliser state.*
- These are states that give a simple spectrum for *Pauli string* operators*:*

$$\mathcal{P}_n = P_1 \otimes P_2 \otimes \ldots \otimes P_N, \quad P_a \in \{\sigma_1^{(a)}, \sigma_2^{(a)}, \sigma_3^{(a)}, I^{(a)}\}$$

Pauli matrix acting on qubit *a*

Identity matrix acting on qubit *a*

- Can make such states by acting on $|0\rangle \otimes |0\rangle \otimes \ldots \otimes |0\rangle$ with Hadamard, phase, CNOT and Pauli gates.

Example: single-qubit stabiliser states

- For a single qubit, one may show that the stabiliser states are:

$$|0\rangle, \quad \frac{1}{\sqrt{2}}\left(|0\rangle \pm |1\rangle\right), \quad \frac{1}{\sqrt{2}}\left(|0\rangle \pm i|1\rangle\right), \quad |1\rangle.$$

- The Pauli string operators in this case are simply $\left(I, \sigma_1, \sigma_2, \sigma_3\right)$.

- For every one of the above states, two Pauli strings have expectation value 1, and the other two give 0.

- As an example, the *Pauli spectrum* (set of expectation values) for $|0\rangle$ is:

$$\mathrm{spec}\left(|0\rangle\right) = \{1, 0, 0, 1\}$$

- This idea of a special family of states with Pauli spectrum values 1 or 0 generalises to *n* qubits.

Stabiliser states for *n* qubits

- More generally, there are 4^n Pauli strings that act on *n* qubits, and the number of stabiliser states is

$$N = 2^n \prod_{k=0}^{n-1} \left(2^{n-k} + 1\right)$$

- This is (6, 60, 1080, 36720) for (1,2,3,4) qubits respectively.

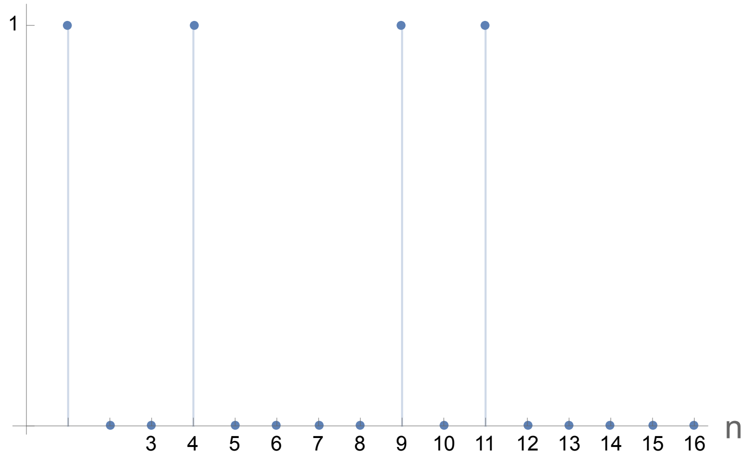
- The Pauli spectrum

$$\mathrm{spec}(|\psi\rangle) = \{\langle\psi|P|\psi\rangle, \quad P \in \mathcal{P}_n\}$$

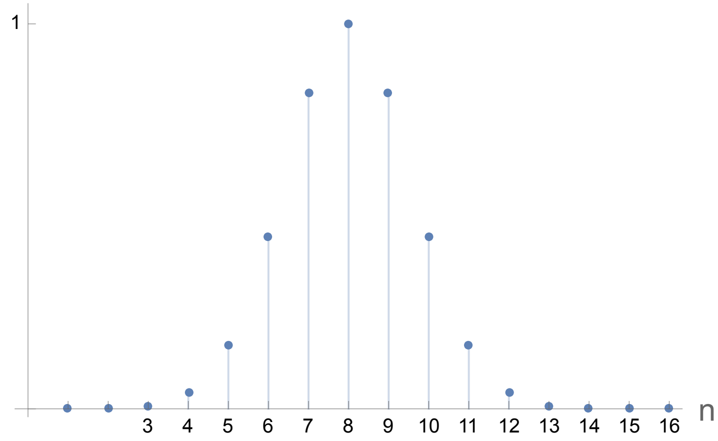
is then such that 2^n values are +1 or -1, and the rest are zero.

Pauli spectra

- We can visualise Pauli spectra as (discrete) distributions.



STABILISER STATE



NON-STABILISER STATE

- It will not yet be clear why these special states are important.
- Let us now see why!

The Gottesman-Knill theorem

- Stabiliser states enter an important result known as the *Gottesman-Knill theorem*:

> For every quantum computer containing stabiliser states only, there is a classical computer that is just as efficient! 😅

- Another way to think about this that quantum circuits containing only CNOT, Hadamard, phase or Pauli gates is no better than a classical computer!
- This is disappointing, given that these are the gates that are easier to realise in real quantum computers.
- Furthermore, stabiliser states include certain **maximally entangled** states.
- Something other than entanglement is needed for efficient quantum computers!

Magic

- The “something else” has been called *magic* in the literature…
- …and basically means “non-stabiliserness” of a quantum state.
- Different definitions exist. We use *Stabilizer Rényi Entropies*: (Leone, Oliviero, Hamma)

$$M_q = \frac{1}{1-q} \log_2\left(\zeta_q\right), \quad \zeta_q \equiv \sum_{P \in \mathcal{P}_n} \frac{\langle \psi | P | \psi \rangle^{2q}}{2^n}$$

- Each (integer) *q* corresponds to a higher moment of the Pauli spectrum.
- The magic is additive, **vanishes** for stabiliser states, and is crucial for making fault-tolerant quantum computers.
- In what follows, examining *q*=2 is enough: the *Second Stabilizer Rényi Entropy (SSRE).*

Magic vs. Entanglement

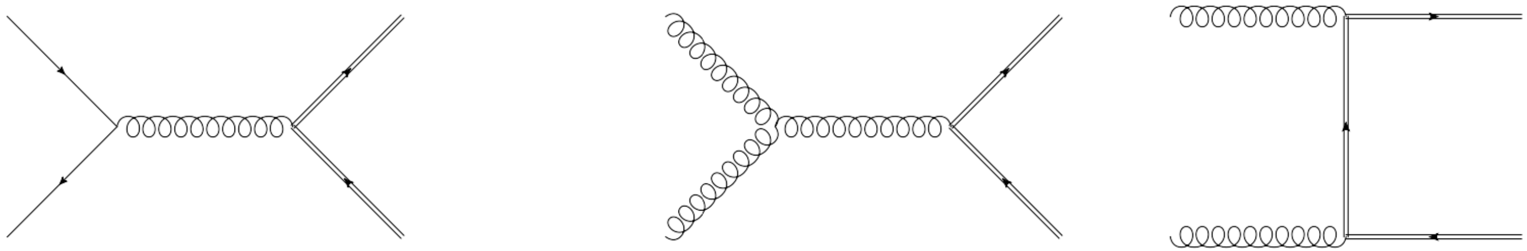
- Magic can be low when entanglement is high.
- Conversely, one can have non-zero magic when entanglement is zero.
- This does not contradict the previous statement about computational advantage.
- Whether quantum computers are faster is a statement about *algorithms* or *circuits.*
- The Gottesman-Knill theorem implies relevant circuits should have entangled states and magic states somewhere.
- The states do not have to be both entangled and magic at all times.
- The relationship between entanglement and magic is an ongoing research area.

Magic at colliders?

- So far, magic has been looked at in condensed matter systems, including in numerical studies.
- It has been studied in nuclear physics (Robin, Savage)…
- …and has even been used to try to explain the origin of spacetime! (Goto, Nosaka, Nozaki).
- High energy colliders such as the LHC have become popular for performing tests of entanglement.
- This suggests they can also be used to study **magic**! 😀
- A good process to look at is that of top quark pair production, which we saw earlier has been used to study entanglement.
- Magic can then be calculated from the relevant **scattering amplitudes**.

Are top quarks magic?

- (Anti-)top quarks are produced in pairs at the LHC…



- …such that the final state is a two-qubit system!
- At LO in the SM there are two different partonic channels.
- The final state is a *mixed state* (superposition of many different *pure states*), where the SM tells us what this is in principle.
- The standard way to talk about mixed states is the *density matrix* formalism.

The density matrix

- Consider a quantum system that can be in states $\{|\psi_i\rangle\}$, with some probability.
- Then the *density matrix* is given by

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$$

Probability of being in state *i*

- Pure states are those whose density matrix has form

$$\rho = |\psi\rangle\langle\psi|$$

- If two or more probabilities are non-zero, we have a *mixed state*.
- Expectation values of operators are given by

$$\langle O\rangle = \sum_i p_i \langle\psi_i|\mathcal{O}|\psi_i\rangle = \mathrm{Tr}[\rho\mathcal{O}]$$

The *R* matrix

- It is known how to calculate the top quark spin density matrix ((Afik, de Nova; Dong, Gonçalves, Kong, Navarro; Fabbrichesi, Floreanini, Panizzo; Aoude, Madge, Maltoni, Mantani, Severi, Boschi, Sioli; Aguilar-Saavedra, Casas).

- First, note that the final (anti-)top pair can be defined by the invariant mass *M* of the top pair, and unit top direction vector ***k***.

- We can then define the *R* matrix for a given partonic channel *I* :

Initial state quantum numbers

$$R^{I\lambda}_{\alpha\beta,\alpha'\beta'}(M,\hat{\boldsymbol{k}}) = \mathcal{A}^{\lambda}_{\alpha\beta}(M,\hat{\boldsymbol{k}}) \left[\mathcal{A}^{\lambda}_{\alpha'\beta'}(M,\hat{\boldsymbol{k}})\right]^{\dagger}$$

Scattering amplitude

(Anti-) top spin indices

- Can then average over initial state quantum numbers λ.

The density matrix

- Once we have the *R* matrix for a given partonic channel, we can easily obtain the density matrix by normalising:

$$\rho^I = \frac{R^I}{\mathrm{Tr}(R^I)}$$

- For proton-proton initial states, one can weight each *R*-matrix by the relevant parton luminosity function:

$$R(M, \hat{\boldsymbol{k}}) = \sum_{I \in \{q\bar{q}, gg\}} L^I(M) R^I(M, \hat{\boldsymbol{k}})$$

- All of the relevant calculations have been carried out in the literature, and it is relatively straightforward to use the results!

Decomposing the density matrix

- On general grounds, the top quark spin *R*-matrix has decomposition:

$$R^I = \tilde{A}^I I_4 + \sum_i \left(\tilde{B}_i^{I+} \sigma_i \otimes I_2 + \tilde{B}_i^{I-} I_2 \otimes \sigma_i + \sum_{i,j} \tilde{C}_{ij}^I \sigma_i \otimes \sigma_j \right)$$

Contribution from partonic channel *I*

Identity matrix

Identity matrix

Pauli matrix

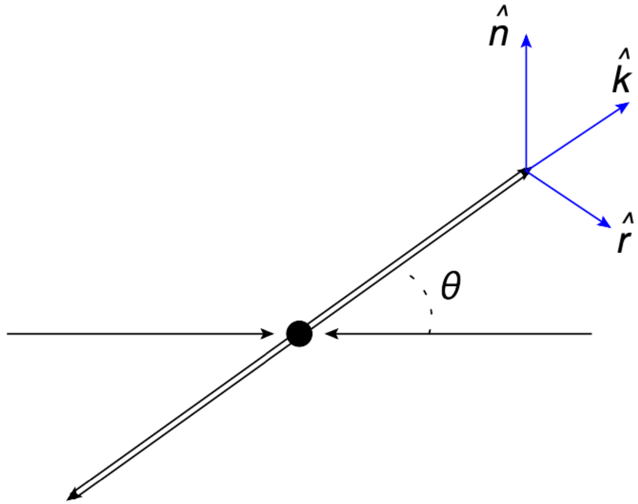
- The *Fano coefficients* $\{\tilde{A}^I, \tilde{B}_i^{I\pm}, \tilde{C}_{ij}^I\}$ depend on the top quark kinematics…
- …as well as the basis relating spin directions (1,2,3) to physical space.
- A common choice is the *helicity basis*.

The helicity basis

- In the helicity basis, one chooses an axis parallel to the top quark direction and two transverse directions (Baumgart, Tweedie).

- Each Fano coefficient is then a function of

$$z = \cos\theta, \quad \beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}}.$$



- In the helicity basis, one chooses an axis parallel to the top quark direction and two transverse directions (Baumgart, Tweedie).

- The coefficients $\tilde{A}^I$, $\tilde{B}^{I\pm}$, $\tilde{C}^I_{ij}$ are related to the total cross-section, (anti-)top polarisation and spin correlations respectively. At LO in the SM:

$$\tilde{B}_i^{I+} = \tilde{B}_i^{I-} = \tilde{C}^I_{nr} = \tilde{C}^I_{nk} = 0, \quad \tilde{C}^I_{ij} = \tilde{C}^I_{ji}$$

Mixed stabiliser states

- We almost have everything we need to define the magic of top quarks!
- First, though, we need to know what a *stabiliser state* means for a mixed state.
- This is defined by a density matrix of form:

Sum over certain Pauli strings

$$\rho = \frac{1}{2^n}\left(I_4 + \sum_{P \in G} \phi_P P \right)$$

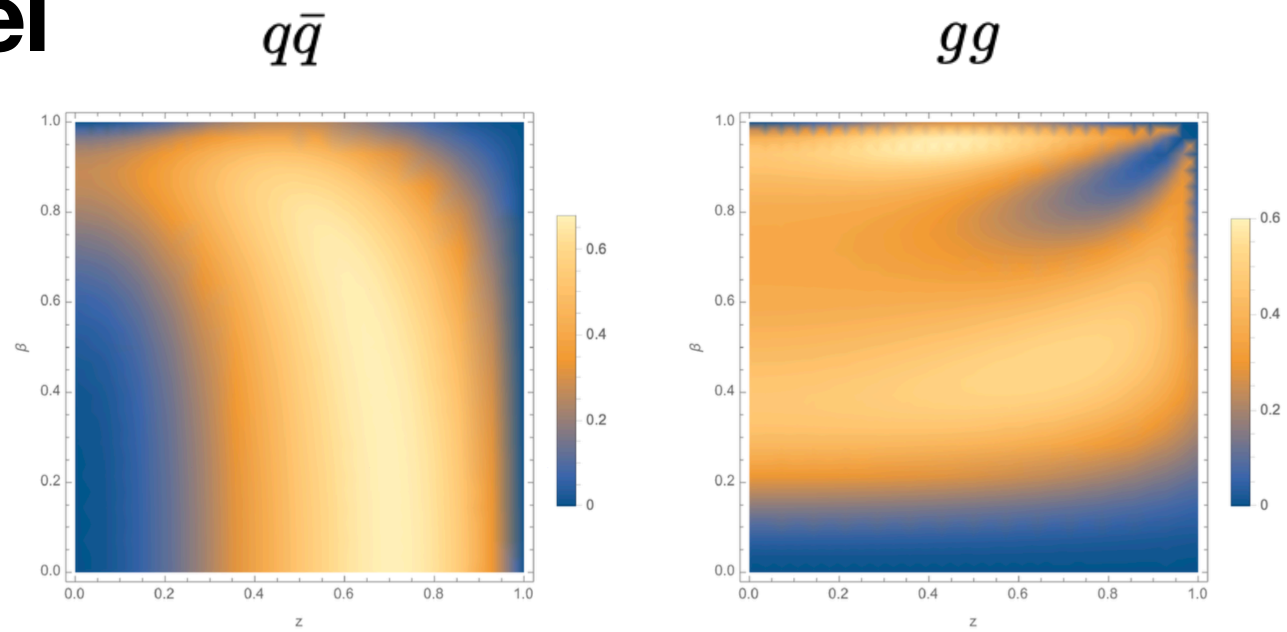
Value of ±1

- Our measure of magic (the second stabiliser Rényi entropy) gets corrected to (Leone, Oliviero, Hamma)

$$\tilde{M}_2(\rho) = -\log_2 \left(\frac{\sum_{P \in \mathcal{P}_n} \mathrm{Tr}^4(\rho P)}{\sum_{P \in \mathcal{P}_n} \mathrm{Tr}^2(\rho P)} \right)$$

Magic: parton level

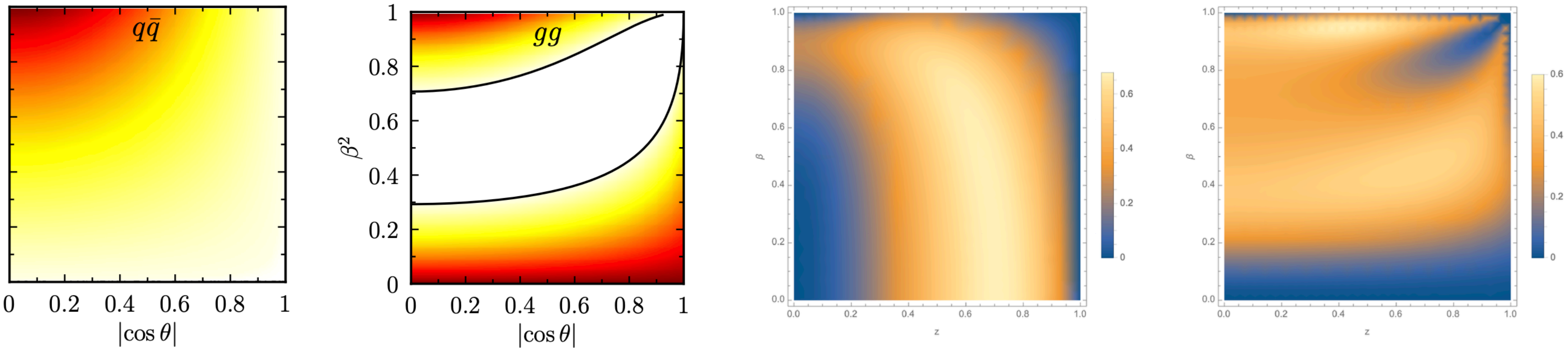
- We can now see how magic top quarks are! 😀
- The magic is clearly non-zero in most of the phase space.



- Unsurprisingly, the amount of magic is different for different partonic channels.
- It also happens to be zero in certain kinematic regions.
- This can be related to known results from the entanglement literature.

Magic vs. entanglement

- It is instructive to compare magic with entanglement (Aoude, Madge, Maltoni, Mantani).



- The left-hand plots show *concurrence* (a measure of entanglement)

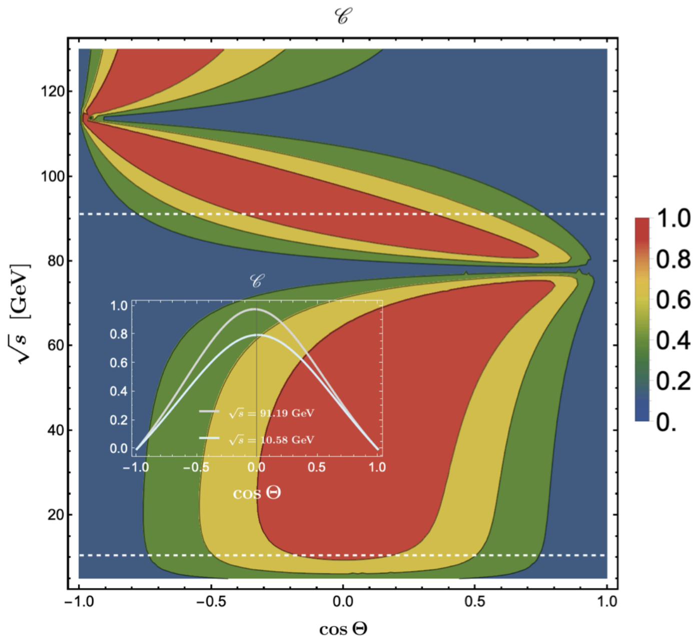
$$\mathscr{C}[\rho] = \max\left(0, r_1 - r_2 - r_3 - r_4\right)$$

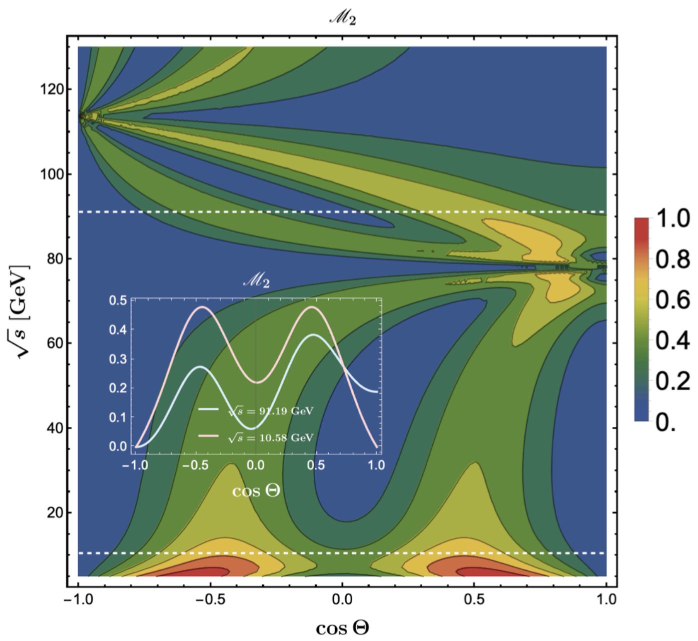
Eigenvalues of spin correlation Matrix

- Entanglement provides very different information to magic.

Magic in other processes

- The role of magic vs. entanglement has been examined for $e^+e^- \to \tau^+\tau^-$ (Fabbrichesi, Low, Marzola).





- Again *concurrence* (entanglement) and *magic* provide very different information.
- The authors provide a measurement of magic in top pair production from CMS data:

$$\mathscr{M}_2 = 0.54 \pm 0.06$$

- They also argue that other quantum information measures can be useful probes of new physics.

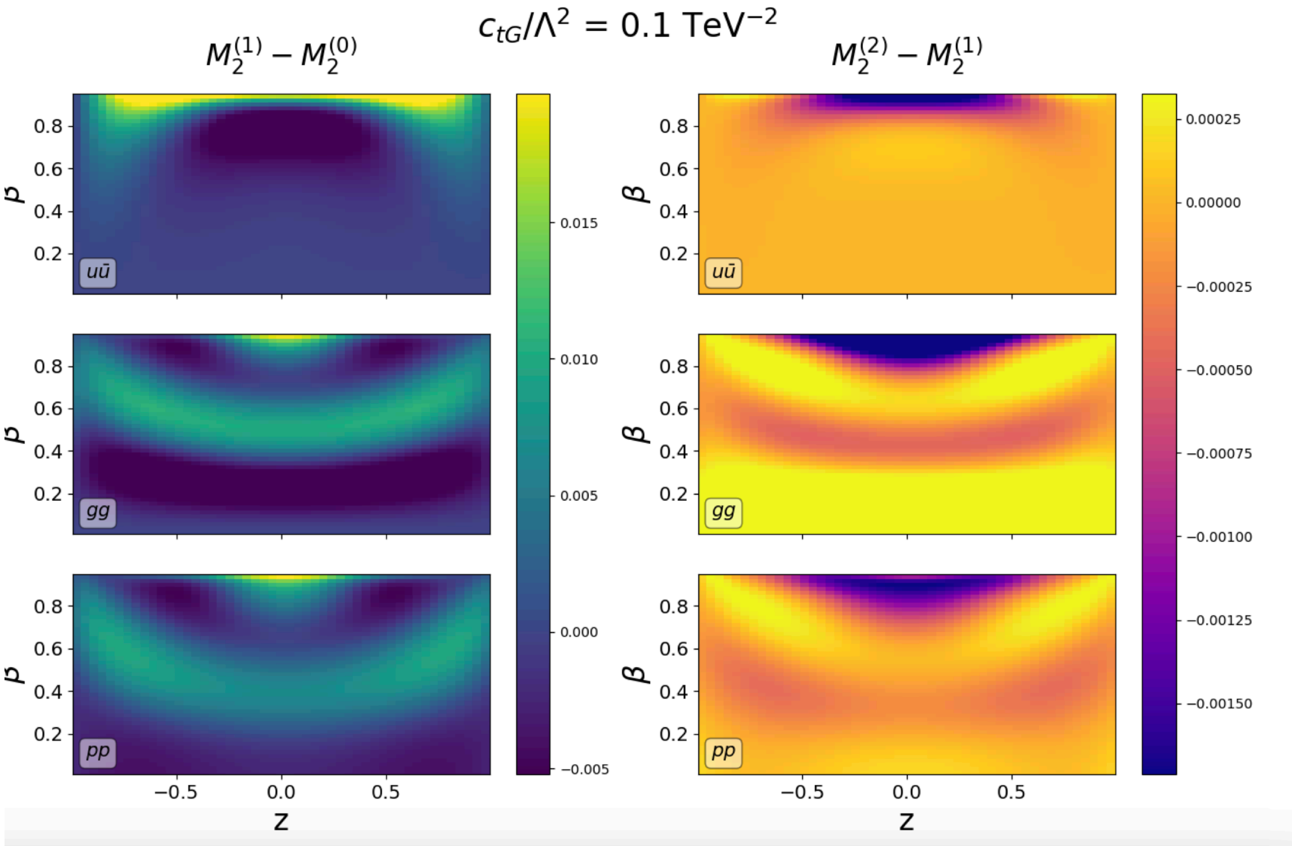
Magic as a probe of new physics

- The profile of magic in top quarks is different in theories beyond the Standard Model of Particle Physics (Aoude, Banks, White, White).

- The plot shows the difference from the Standard Model, for a particular new physics scenario.

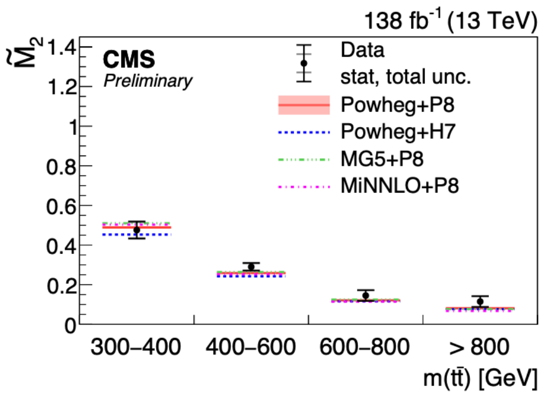
- We can use these differences to find efficient ways of finding new physics.

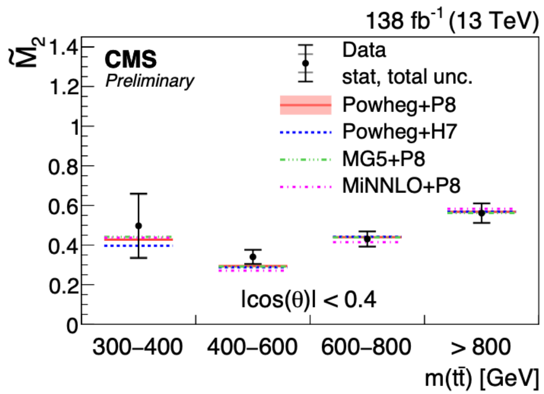
- Other QI measures (e.g. fidelity, trace distance) are also useful (Fabbrichesi, Low, Marzola).

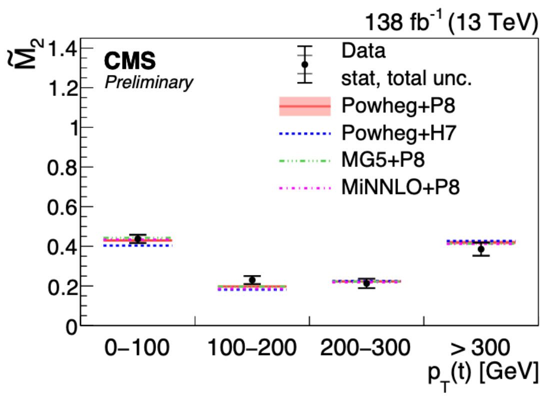


Magic at the LHC

- Recently, the CMS experiment at CERN reported a first measurement of magic in top quarks! 😀







- The data show the amount of **magic** for different ranges of top quark velocity / angle.
- The analysis was led by Regina Demina, with her PhD student Alan Herrera at the University of Rochester.
- Other LHC experiments (e.g. ATLAS) are also interested.

Theoretical studies of magic: QED

- There is a growing series of papers looking at scattering in different theories, and classifying how much magic is generated.

- An upper bound has been proposed for magic in two-qubit scattering (Liu, Low, Yin):

$$M_2 \leq \log\left(\frac{16}{7}\right) \simeq 0.827$$

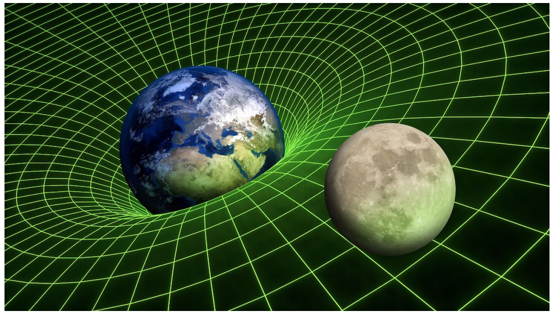
- The same authors looked at various scattering processes in Quantum Electrodynamics (QED).

- The found that QED scattering is not very efficient at generating large amounts of magic on average.

From gluons to gravitons

- Another interesting case study is to look at gluon scattering.
- Starting from a given 2-qubit initial state with fixed helicity (spin), how much magic do we get in the final state?
- We can derive similar results for (quantum) General Relativity using known relations with gluons (Kawai, Lewellen, Tye).
- This is a novel example of a pair of 2-qubit systems, differing in the **spin** of the qubit.

Gluon (strong force), spin 1

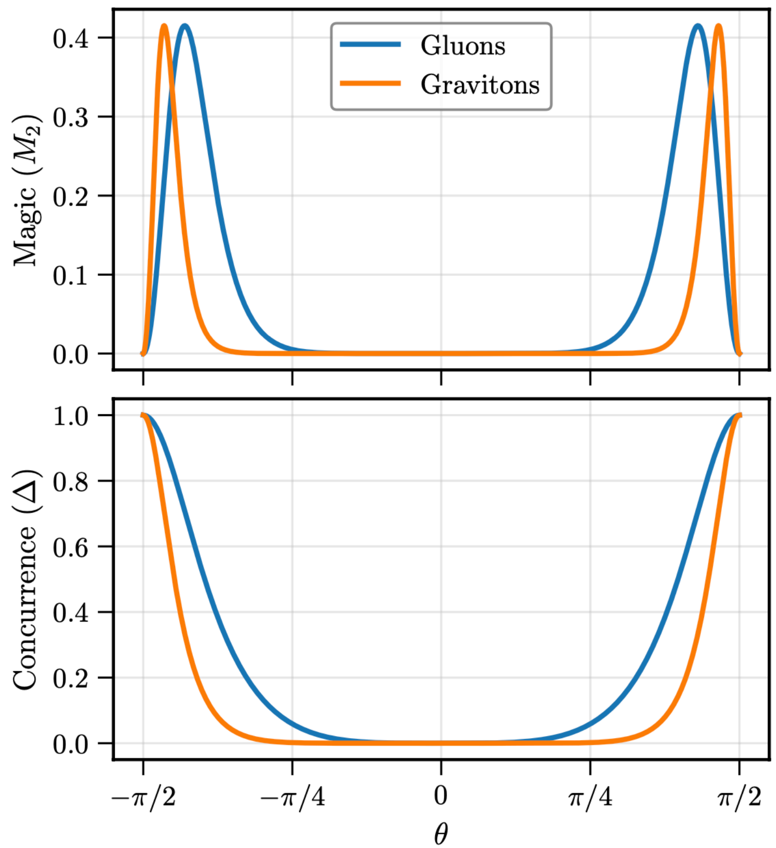


Graviton (gravity), spin 2

Gluons and gravitons: results

- Can evaluate the final-state magic for different scattering angles θ.
- For the |+-> initial state, the magic gets more concentrated for gravitons…
- …and the same is true for entanglement.
- The maximum magic is not near the upper bound of 0.827.
- Starting with *any* possible stabiliser state, we find the overall maximum magic is:

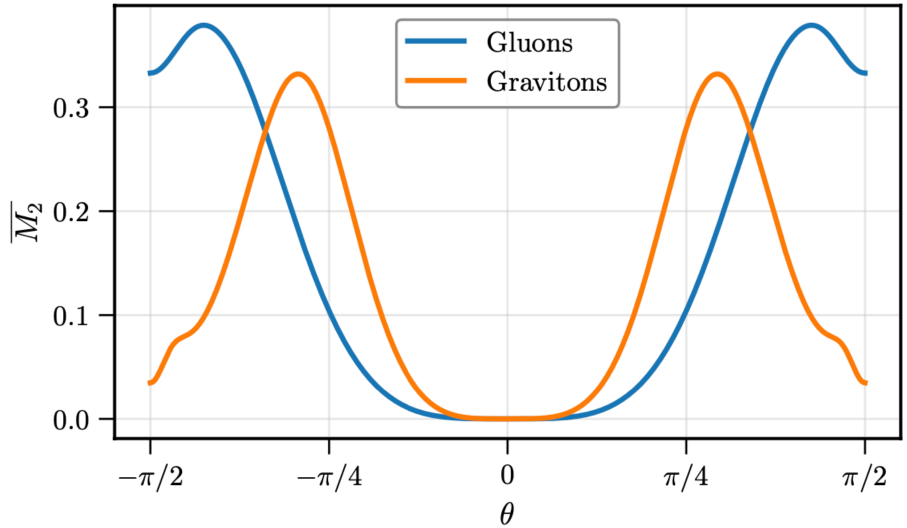
$$M_2^{\max}\Big|_{\mathrm{YM}} = 0.529635, \quad M_2^{\max}\Big|_{\mathrm{grav.}} = 0.415049.$$



Magic power

- It seems that there is typically less magic in gravity than for gluons.
- To make this precise, we can calculate the *magic power* (average final state magic, over all initial stabiliser states).
- The integrated results are:

$$\int_0^{\pi/2} \overline{M_2}(\theta)\Big|_{\text{YM}} = 0.245,$$
$$\int_0^{\pi/2} \overline{M_2}(\theta)\Big|_{\text{grav}} = 0.208.$$

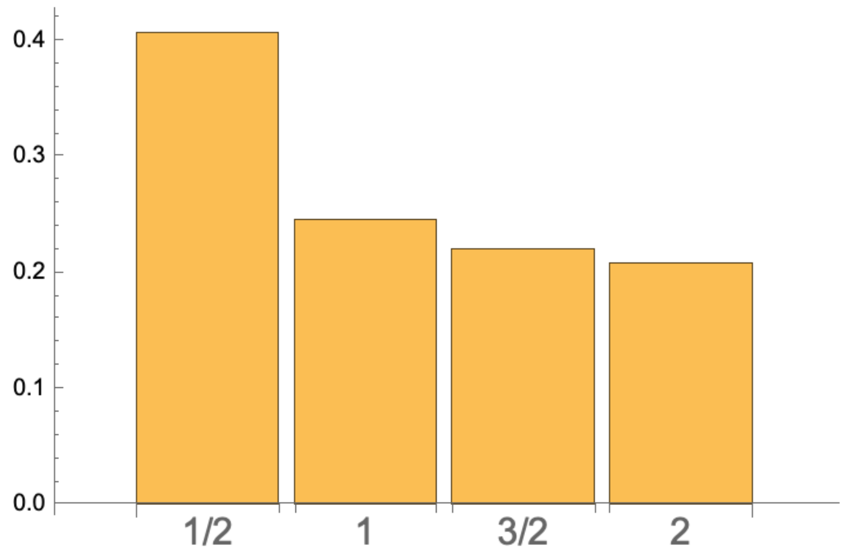


- Perhaps it is true that more spin = less magic?

Gluinos and gravitinos

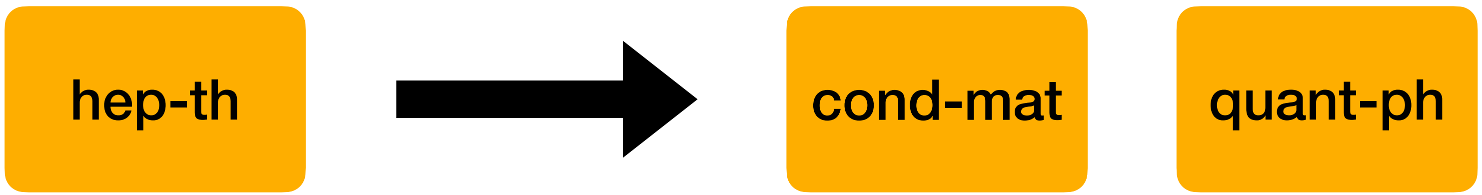
- To investigate this, we can consider supersymmetric extensions of gluon theory and General Relativity.
- Supersymmetry adds particles like the gluon and graviton, but will spin 1/2 (*gluinos*), and 3/2 (*gravitinos*).
- This gives us two more data points for magic power vs. qubit spin!

- Indeed the amount of magic decreases with increasing qubit spin.
- May be relevant for condensed matter systems near critical points?



Scattering amplitudes as tools

- Differences in magic between gravity and Yang-Mills arise from the relevant scattering amplitudes…
- …which are in turn related by KLT relations, or the **double copy** at higher loop orders (Bern, Carrasco, Johansson).
- Thus, formal ideas from the scattering amplitudes literature have a role to play in studying magic in different quantum systems.
- The lessons may then be portable to other fields (condensed matter, QI).



Non-local magic and AdS / CFT

- Magic is a basis-dependent quantity…
- …and a variant has been proposed that gets round this problem.
- For a bipartite system, one minimises the amount of magic over all independent unitary transformations of each part.
- This is called **non-local magic** (Qian, Wang).
- It has recently been argued to be the holographic dual (in boundary CFTs) of gravitational back-reaction in the bulk (Cao, Cheng, Hamma, Leone, Munizzi, Oliviero).
- It would be interesting to know what other studies are possible in this area…
- …especially given the many applications of CFT in condensed matter!

Conclusions

- Magic is a property of quantum states that distinguishes computational advantage over classical computers.
- It has recently become interesting for high-energy physics.
- QI measures may help find new physics, or explain structures in our theories.
- HEP systems provide new playgrounds for investigating magic, that may be interesting for other fields.
- Scattering amplitudes might be useful for probing magic in general quantum systems.
- This topic has just reached **hep-th**, so there are lots of possible ideas! 😀