

Non-Lorentzian Holography and Near-BPS Physics in String Theory

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Based on [2410.17074](#), [W.I.P.](#) w/ Neil Lambert (see also [2401.14955](#), [2405.06552](#))
[2502.07969](#) w/ Neil Lambert & Eric Bergshoeff



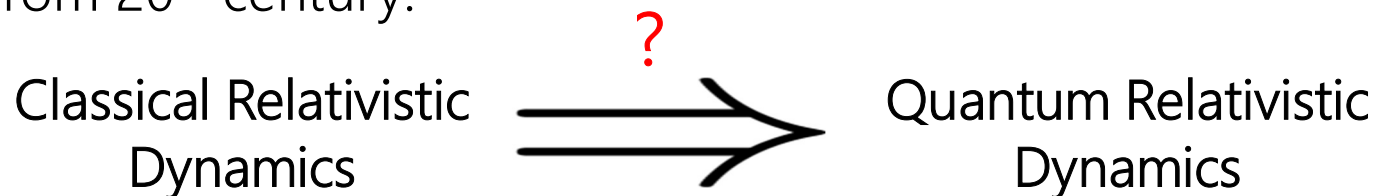
Motivation

Would like to understand string theory non-perturbatively: this is famously hard!

No accepted microscopic formulation, though many proposals (e.g. string field theory, large-N BFSS, holography, etc.)

Could argue we're trying to do too much at once: plethora of non-perturbative dynamical *relativistic* objects, need to account for their production and interactions

Analogy from 20th century:



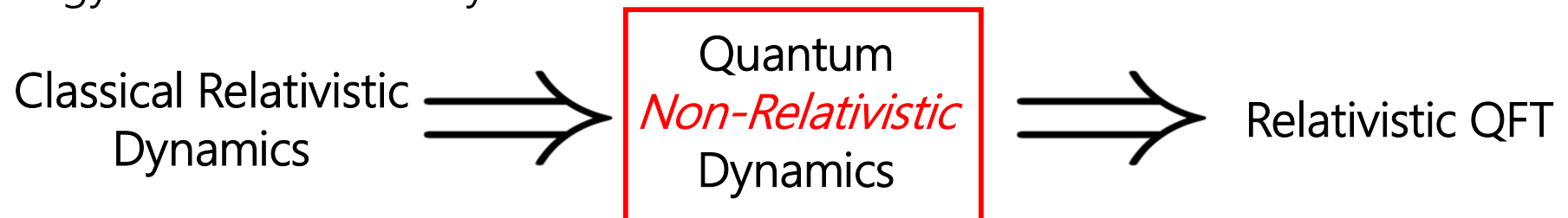
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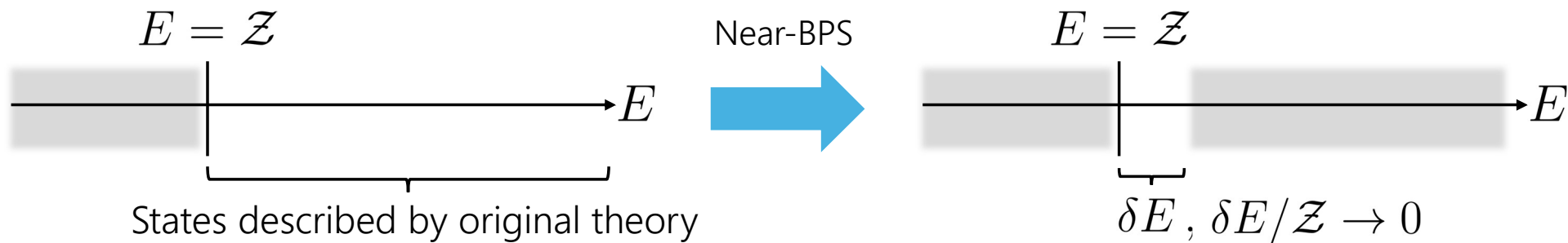


BPS Decoupling Limits

Goal: Find self-contained corners of string theory built whose dynamics is governed by a single type of brane

Idea: Isolate states near BPS bound,

Harmark,
Orselli '14



Infinitesimal excitations above bound \rightarrow fixed number of branes

Physically corresponds to considering *non-Lorentzian* BPS branes

Warm Up: Free Scalar Field

To get a feel for these limits, let's consider a free complex scalar field:

$$S = -\frac{1}{2} \int d^{d+1}x \sqrt{-g} (g^{\mu\nu} \partial_\mu \bar{\varphi} \partial_\nu \varphi + M^2 \bar{\varphi} \varphi)$$

To project onto low energy states we must consider the long-timescale limit, i.e. we work on

$$g = c^2 \left(- \cancel{c^2 dt^2} + dx^i dx^i \right) \quad \text{Deformation parameter, not 'physical' speed of light}$$

and take $c \rightarrow \infty$ limit. Problem: As it stands, this kills all dynamics!

Solution is well known: couple theory to a background U(1) field and rescale,

$$\varphi = \sqrt{c} \phi, \quad A = c dt$$

Warm Up: Free Scalar Field

The action is then

$$S = \frac{1}{2} \int dt d^d x \left(c^{-2} \partial_0 \bar{\phi} \partial_0 \phi + 2iq \bar{\phi} \partial_0 \phi - \partial_i \bar{\phi} \partial_i \phi - (M^2 - q^2) c^2 \bar{\phi} \phi \right)$$

Introduces divergent term that cancels if we take extremal limit,

$$M = q$$

Key point: In order to take a non-Lorentzian limit of particles, we must

1. Rescale the underlying geometry to define a static particle worldline and transverse directions
2. Couple to an extremal U(1) field to cancel off divergent rest energy

BPS Decoupling Limits of D-Branes

We now do the same in string theory. However, can't arbitrarily couple objects to background fields: need extremality w.r.t. a physical U(1) field, i.e. need to work with BPS branes. Consider Dp-brane with action (ignoring B-field)

$$S_p = -T_p \int d^{p+1}\xi e^{-\phi} \sqrt{-\det (P[G] + 2\pi\alpha' F)} + T_p \int C_{p+1}$$

Aim: after gauge-fixing, want to recover the usual low-energy D-brane action

$$S_p \rightarrow -\frac{(2\pi\alpha')^2 T_p}{2} \int d^{p+1}x \left(\partial_a \Phi^I \partial^a \Phi^I + \frac{1}{2} F_{ab} F^{ab} \right)$$

$\Phi^I = 2\pi\alpha' X^I$

Need to rewrite fields in a $SO(1, p) \times SO(9 - p)$ -invariant way that automatically deals with divergent terms and leads to action above!

BPS Decoupling Limits of D-Branes

After some inspiration, we find

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$$g_{10} = c^2 \eta_{p+1} + c^{-2} \delta_{9-p}, \quad \phi = (p-3) \ln c, \quad C_{p+1} = c^{p+1} \varepsilon_{p+1}$$

Observation: This is equivalent to the D-brane's supergravity solution with harmonic function $\mathcal{H}_p = c^{-4}$

However! String theory is a theory of gravity: need to extend this to curved geometries. For metric, this is

$$\tau_{\mu\rho} \mathcal{E}^{\rho\nu} = 0 \leftarrow \begin{aligned} g_{\mu\nu} &= c^2 \tau_{\mu\nu} + c^{-2} \mathcal{E}_{\mu\nu}, \\ g^{\mu\nu} &= c^2 \mathcal{E}^{\mu\nu} + c^{-2} \tau^{\mu\nu} \end{aligned} \rightarrow \text{Projective inverses}$$

Dp-Brane Newton Cartan Geometry

Generalisation to Any BPS Brane

This gives us a recipe for a near-BPS limit of any brane in string/M-theory:

1. Take supergravity solution, replace harmonic function with a negative power of \mathcal{C}
2. Promote metric components to brane-like Newton Cartan fields, write divergent form-fields in terms of volume forms
3. Allow additional finite dilaton (if in 10d) and form-field terms

Example: M2 and M5-branes in 11d,

M2-brane Limit:

$$g_{11} = c^2 \tau_3 + c^{-1} \mathcal{E}_8 ,$$
$$C_3 = -c^3 \varepsilon_3 + c_3$$

M5-brane Limit:

$$g_{11} = c^2 \tau_6 + c^{-4} \mathcal{E}_5 ,$$
$$C_6 = c^6 \varepsilon_6 + c_6$$

Non-Lorentzian Supergravity

Want to understand the physics of these limits! First step: look at long wavelength approximation, i.e. underlying supergravity theory

Limit can be taken at level of action or equations of motion: for consistency, we need both to agree*

Let's be explicit: example we'll study is M5-brane limit of 11d supergravity,

$$S_{11} = \frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} R(g) - \frac{1}{4\kappa_{11}^2} \int \left(F_4 \wedge *F_4 - \frac{1}{3} C_3 \wedge F_4 \wedge F_4 \right)$$

Expectation: Gravitational and gauge forces balanced for BPS states, so *small* universally attractive force for states infinitesimally above bound → Newtonian-esque gravity! Should find a form of Poisson equation

Non-Lorentzian Supergravity

At level of action: must use metric/vielbein and 3-form as variables, so only possible structure respecting 5-brane symmetries is to leave 3-form unscaled,

$$E_{\mu}^M = \{c \tau_{\mu}^A, c^{-2} e_{\mu}^a\} , \quad C_3 = c_3$$

Plugging decomposition into action, find divergent terms:

$$S_{11} = c^{12} S_2 + c^6 S_1 + S_0 + O(c^{-6})$$

Fortunately, after rearranging divergent pieces are squares,

$$S_{1,2} = - \int d^{11}x \, \Omega \mathcal{C}_{1,2}^2$$

Measure formed from $\{\tau_{\mu}^A, e_{\mu}^a\}$


Non-Lorentzian Supergravity

Can then regulate these terms using Hubbard-Stratonovich fields,

$$c^{2\alpha} \int d^{11}x \, \Omega \, \mathcal{C}_{1,2}^2 \Rightarrow \int d^{11}x \, \Omega \left(2\lambda_{1,2} \cdot \mathcal{C}_{1,2} - c^{-2\alpha} \lambda_{1,2}^2 \right)$$

Non-Lorentzian Supergravity

Can then regulate these terms using Hubbard-Stratonovich fields,


$$c^{2\alpha} \int d^{11}x \Omega \mathcal{C}_{1,2}^2 \Rightarrow \int d^{11}x \Omega \left(2\lambda_{1,2} \cdot \mathcal{C}_{1,2} - c^{-2} \mathcal{C}_{1,2}^2 \right)$$


Impose constraints on theory: for the M5-brane limit, we have

$$f_{abcd} = 0 ,$$

$$(d\tau^A)_{ab} = \frac{1}{6} \epsilon_{abcde} f^{cdeA}$$

Obstruction to
'absolute' 5-
brane foliation



Had to add additional fields to the description here – is this consistent with e.o.m. expansion? (Will come back to this!)

Non-Lorentzian Supergravity

Add this to finite part of the action to get well-defined theory, but no Poisson equation: what's going wrong?

Answer: additional gauge symmetry emerges after limit! Local dilatation symmetry,

$$\tau^A \rightarrow e^{\omega(x)} \tau^A, \quad e^a \rightarrow e^{-2\omega(x)} e^a$$

Morally equivalent to
local rescaling of \mathcal{C}

Noether's 2nd theorem: implies additional relation between e.o.m.,

$$2e_a^\mu \mathbb{E}_\mu^a + \tau_\mu^A \mathbb{E}_A^\mu = 0$$

Fewer e.o.m. than relativistic theory, so must proceed via equations of motion: projection of leading order e.o.m. vanishes, so can go to subleading order to find additional scalar equation

Non-Lorentzian Supergravity

Easiest to handle e.o.m. expansion by introducing on-shell dual field,


$$G_7 = dC_6 - \frac{1}{2}C_3 \wedge F_4 \Rightarrow G_7 = *F_4$$

Strategy: Project equations along all combinations of τ^A and e^a , and extract leading terms

When applied to the duality relation, recover constraints imposed by HS fields!
Also able to identify these with finite components of 7-form g_7

Now look at Einstein equation: find expected cancellation, with subleading equation

$$\nabla_\mu (e_a^\mu g_a) + 2(d\tau^A)_{aA} g_a = \text{geom.} + \text{flux}$$

$g_a \propto \epsilon^{ABCDEF} g_{aABCDEF}$  5-brane Poisson equation!

Non-Lorentzian Brane Solutions

Can construct solutions by inserting deformation parameter in relativistic solutions and taking limit

How can we ensure that we get something sensible after limit? Answer: Arrange M5-brane limit and backreacting brane in $\frac{1}{4}$ -BPS configuration

Example: M2-brane,

$$g_{11} = \mathcal{H}_5^{-1/3} \left(\mathcal{H}^{-2/3} \eta_2 + \mathcal{H}^{1/3} d\sigma^I d\sigma^I \right) + \mathcal{H}_5^{2/3} \left(\mathcal{H}^{-2/3} dx^2 + \mathcal{H}^{1/3} dX^M dX^M \right)$$
$$C_3 = (\mathcal{H}^{-1} - 1) dt^0 \wedge dt^1 \wedge dx$$

Non-Lorentzian Brane Solutions

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How can we ensure that we get something sensible after limit? Answer: Arrange M5-brane limit and backreacting brane in $\frac{1}{4}$ -BPS configuration

Example: M2-brane, Take this to be T^4 ,
smear M2-branes

$$g_{11} = c^2 \left(\mathcal{H}^{-2/3} \eta_2 + \mathcal{H}^{1/3} \overbrace{d\sigma^I d\sigma^I} \right) + c^{-4} \left(\mathcal{H}^{-2/3} dx^2 + \mathcal{H}^{1/3} dX^M dX^M \right)$$

$$C_3 = (\mathcal{H}^{-1} - 1) dt^0 \wedge dt^1 \wedge dx$$

Harmonic function for solution is

$$\mathcal{H} = 1 + \frac{R^2}{r^2}$$

Non-Lorentzian Brane Solutions

Smearing ensures C -independence of M2-brane flux,

$$N_{M2} \propto \int_{S^3_\infty \times T^4} *F_4$$

Taking near-horizon limit and using local dilatation,

$$\tau_6 = \boxed{\frac{r^2}{R^2} \eta_2} + d\sigma^I d\sigma^I, \quad \mathcal{E}_5 = dx^2 + \boxed{\frac{R^2}{r^2} dr^2} + R^2 g_{S^3}$$

Non-Lorentzian
analogue of AdS_3

Same idea holds in general: get well-defined brane solution (i.e. finite flux) if we start with a $\frac{1}{4}$ -BPS intersecting brane set-up and smear over coordinates in \mathcal{T} transverse to backreacting brane

Beyond Supergravity?

Seen that limits lead to well-defined theories of Newtonian-like gravity in the IR: can we go beyond this and take limit of full QG theory?

Best understood case is non-relativistic string limit: underlying (bosonic) string theory is Gomis-Ooguri string, i.e. left+right moving $\beta\gamma$ system + D-2 transverse scalars on flat background

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Two perspectives,

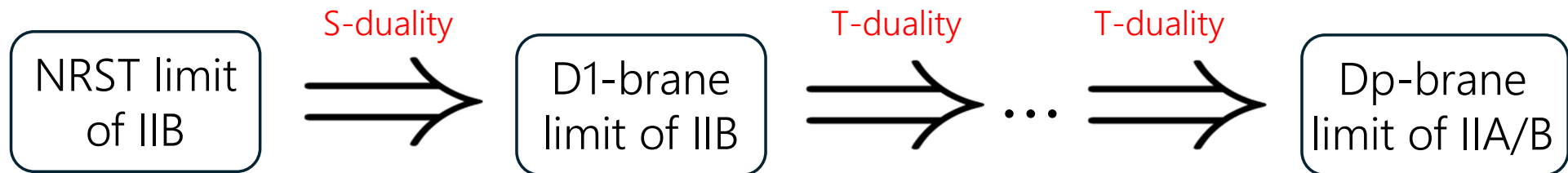
1. Compute scattering amplitudes in flat background – done explicitly at tree-level and one-loop, UV-finite results
2. Couple worldsheet to background fields (i.e. turn on marginal deformations) and compute beta functions – at one-loop, agree with NRST limit of supergravity (including Poisson equation)

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UV completion of non-Lorentzian gravity

Beyond Supergravity?

Can go further: near-BPS limits obey usual string theory dualities, so applying in sequence gives



Claim: All near-BPS limits of string/M-theory lead to UV-complete theories of non-Lorentzian quantum gravity

Microscopic dynamics given by interacting near-BPS (i.e. low energy) branes: understood in various cases, e.g. Dp-brane limits on a flat background theory are described by $U(N)$ $(p+1)$ -dim. MSYM

Aside: How Does AdS/CFT Fit Into This?

So far we've said nothing about the most famous example of a decoupling limit – the duality between $\mathcal{N} = 4$ SYM and string theory on $AdS_5 \times S^5$.

Can we understand this from our discussion so far?

Consider the IIB supergravity solution,

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$$g_{10} = \frac{r^2}{R^2} \eta_4 + \frac{R^2}{r^2} \delta_6, \quad C_4 = \frac{r^4}{R^4} \varepsilon_4$$

Defining a 'holographically-running speed of light' $\tilde{c}(r) = r/R$, this is

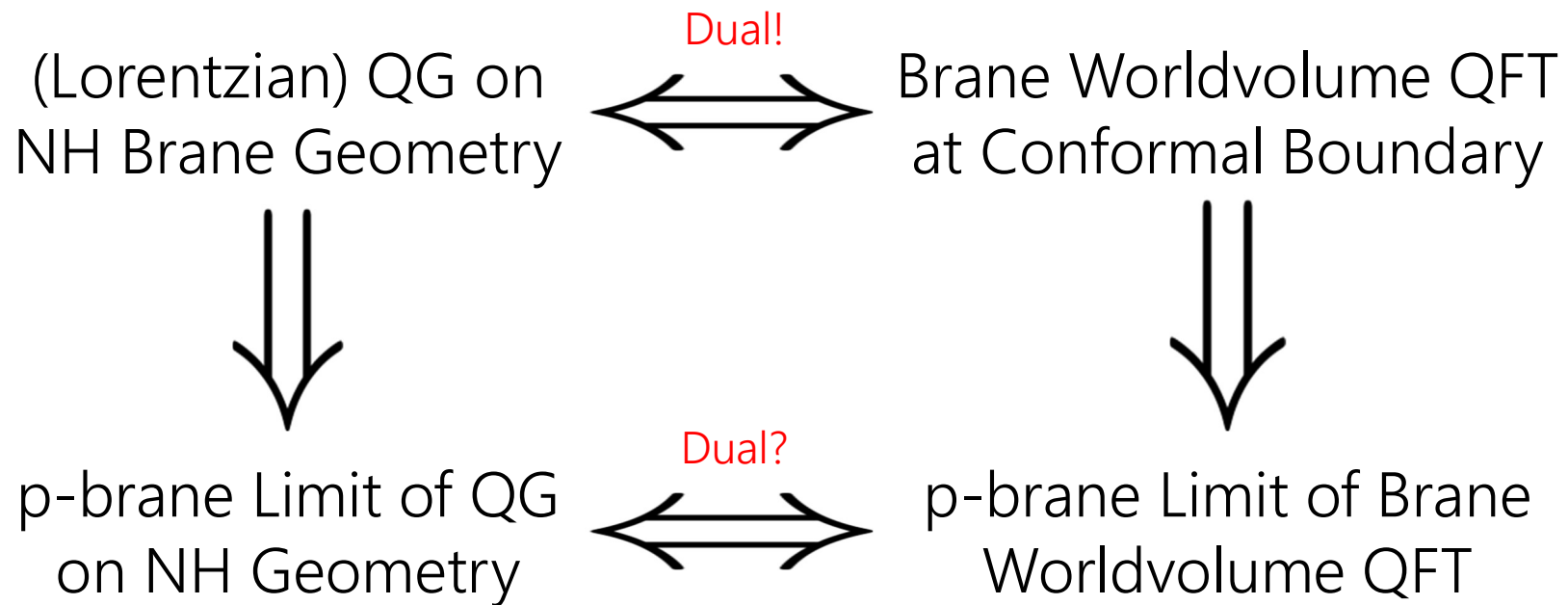
$$g_{10} = \tilde{c}^2 \eta_4 + \tilde{c}^{-2} \delta_6, \quad C_4 = \tilde{c}^4 \varepsilon_4$$

$$r \rightarrow \infty \Rightarrow \tilde{c} \rightarrow \infty \Rightarrow \text{D3-brane limit at asymptotic boundary}$$

Novel perspective on why low-energy brane QFT appears at boundary!

Non-Lorentzian Holography

How can we probe properties of limits? Use holography,

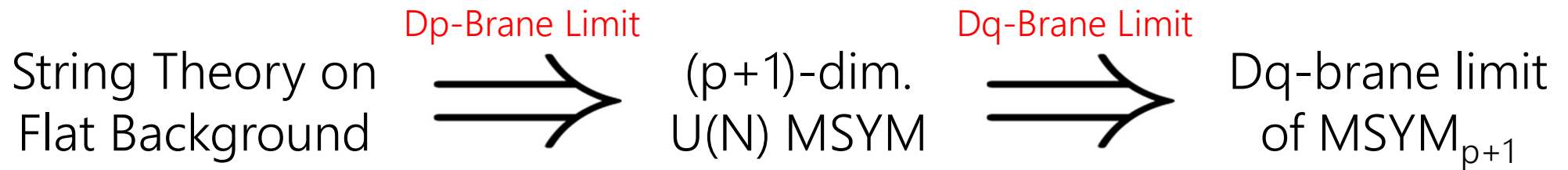


If we believe holography holds at quantum level, both sides should match!

Single limit of string theory on curved background vs double limit on flat background

Non-Lorentzian Holography

Focus on D-brane decoupling limits. Consider Dq-brane limit of Dp-brane,



First takes us to near-BPS Dp-branes, second takes us to near-BPS Dq-branes *within* near-BPS Dp-branes: in other words, taking both localises us onto near-BPS Dp-Dq bound states

Can consider reversing limits: physically should pick out same bound-state dynamics, so can propose the equality

$$\text{Dq-brane limit of MSYM}_{p+1} = \text{Dp-brane limit of MSYM}_{q+1}$$

Example: D0-D4 Bound States

Start with D0-D4 states: take D4-brane limit first. Worldvolume QFT on D4-branes is 5d $\mathcal{N} = 2$ SYM,

$$S_{D4} = -\frac{1}{4g^2} \text{tr} \int d^5x \left(F_{\mu\nu} F^{\mu\nu} - 2D_\mu Y^M D^\mu Y^M + [Y^M, Y^N]^2 \right)$$

Need to translate D0-brane limit of flat background into QFT scaling:

Decomposition of Metric $\{t, x^i\} \rightarrow \{c t, c^{-1} x^i\}$, $Y^M \rightarrow c^{-1} Y^M$

Dilaton Scaling $\Rightarrow g^2 \rightarrow c^{-3} g^2$

Divergent 1-form Field $\Delta S_{D4} = \frac{c^4}{2g^2} \text{tr} \int dt \wedge F \wedge F$

Example: D0-D4 Bound States

Substituting into action gives

$$S_{D4} = \frac{1}{2g^2} \text{tr} \int dt d^4x \left(- \underbrace{\frac{c^4}{2} (F - *F)^2}_{\text{Divergent Squared Term!}} + F_{0i} F_{0i} - D_i Y^M D_i Y^M \right)$$

Divergent Squared Term!

As in supergravity limit, tame with Hubbard-Stratonovich transformation:
imposes YM instanton equation

$$F = *F$$

YM kinetic term reduces to moduli space kinetic term

$$\text{tr} \int dt d^4x F_{0i} F_{0i} = \int dt g_{\alpha\beta}(m) \dot{m}^\alpha \dot{m}^\beta$$

Hyper-Kähler metric
on moduli space

(Supersymmetric) QM on Instanton Moduli Space

Example: D0-D4 Bound States

Now look at supergravity solution: using method discussed earlier, find

$$g_{10} = -c^2 \mathcal{H}^{-1/2} dt^2 + c^{-2} \left(\mathcal{H}^{-1/2} dx^i dx^i + \mathcal{H}^{1/2} dY^M dY^M \right)$$
$$e^\phi = c^{-3} \mathcal{H}^{-1/4}, \quad C_1 = c^4 dt, \quad C_5 = (\mathcal{H}^{-1} - 1) dt \wedge dx^1 \wedge \dots \wedge dx^4$$

for D4-brane in terms of harmonic function

$$\mathcal{H} = 1 + \frac{R^3}{r^3}$$

As it stands, this is messy: however, D0-brane theory has local dilatation symmetry which we can use to trivialise dilaton,

$$c \rightarrow c \mathcal{H}^{-1/12}$$

Example: D0-D4 Bound States

Applying this to solution and taking near-horizon limit gives

$$g_{10} = -c^2 \frac{r^2}{R^2} dt^2 + c^{-2} \left(\frac{r}{R} dx^i dx^i + \frac{R^2}{r^2} dr^2 + R^2 g_{S^3} \right)$$

Non-Lorentzian *AdS*-like spacetime

$$C_5 = \frac{r^3}{R^3} dt \wedge dx^1 \wedge \dots \wedge dx^4$$

Possesses an emergent $z=2$ Lifshitz scaling symmetry,

$$t \rightarrow \lambda^2 t, \quad x^i \rightarrow \lambda x^i, \quad r \rightarrow \lambda^{-2} r$$

Actually enhances to full Schrödinger symmetry

Same symmetry emerges after limit in gauge theory description due to presence of homothetic killing vector on moduli space

Example: D1-D3 Bound States

Can do the same with T-dual setup: D1-D3 bound states. Can either take limit starting from $\mathcal{N} = 4$ or dimensionally reduce D0-D4 action

Find SQM on the moduli space of BPS monopole equations,

$$F_{ij} = \epsilon_{ijk} D_k X$$

which require $\langle X \rangle \neq 0$ to have non-trivial solutions: corresponds to monopole mass scale

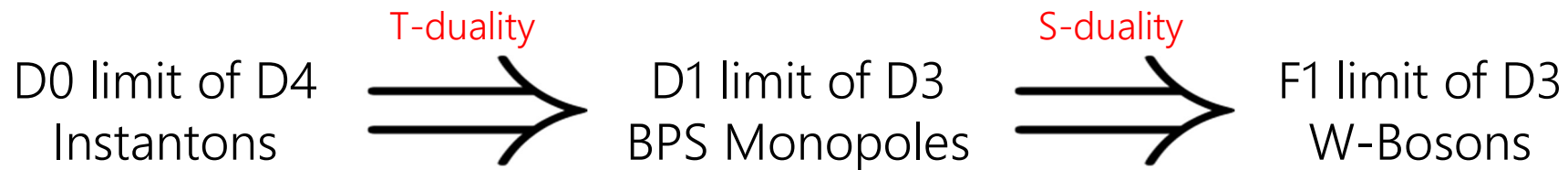
Physically corresponds to D3-brane separation: monopoles are D1-branes stretched across interval [hep-th/9608163](https://arxiv.org/abs/hep-th/9608163)

Conformal symmetry of $\mathcal{N} = 4$ broken by VEV: also seen in dual supergravity solution

Non-Relativistic Strings and Galilean YM

D-brane limits interesting, but hard to quantise: in gauge theory description have Lagrange multipliers, and in moduli space description need explicit understanding of metric

Look for limit in which dynamics arise from perturbative QFT states: a good choice is to look at non-relativistic string limit of D-brane QFT,



QFT limit goes through using same steps as before: novelty here is that transverse scalars split into two groupings and B-field induces shift of connection,

$$\{X, Y^M\} \rightarrow \{cX, Y^M\} , \quad A_0 \rightarrow A_0 + c^2 X$$

Non-Relativistic Strings and Galilean YM

Resulting theory is Galilean Yang-Mills,

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$$S_{GYM} = \frac{1}{2g^2} \text{tr} \int dt d^p x \left((D_0 X)^2 - 2D_i X F_{0i} - \frac{1}{2} F_{ij} F_{ij} - 2i D_0 Y^M [X, Y^M] \right. \\ \left. - D_i Y^M D_i Y^M + \frac{1}{2} [Y^M, Y^N]^2 \right)$$

How do we see dynamics? Restrict to SU(2) theory and work on Coulomb branch, $\langle X \rangle = v\sigma^3/2$. Expanding other scalars as

$$Y^A = y^A \sigma^3 / \sqrt{2} + \omega^A \sigma^+ + \bar{\omega}^A \sigma^-$$

gives quadratic terms

$$S_{quad.} = \int dt d^p x \left(\underbrace{2iv \bar{\omega}^A \partial_0 \omega^A - \partial_i \bar{\omega}^A \partial_i \omega^A}_{\text{Non-relativistic particle of mass } M = v} - \frac{1}{2} \overbrace{\partial_i y^A \partial_i y^A}^{\text{Non-Dynamical Field}} \right)$$


Non-Relativistic Strings and Galilean YM

General structure of theory:

- Galilean electrodynamics + real scalars with only spatial derivatives (i.e. abelian limit of GYM action)
- Massive charged non-relativistic particles
- Fermions (give supersymmetric theory)

Interesting case is $p = 2$, describing low-energy physics of a D2-brane in NRST: observe Schrödinger symmetry, with scaling

$$\{t, x^i\} \rightarrow \{\lambda^2 t, \lambda x^i\}, \quad \boxed{X \rightarrow X}, \quad Y^M \rightarrow \lambda^{-1} Y^M$$

VEV Preserved! 

Same symmetries observed in near-horizon limit of D2-brane on transverse circle in NRST: does this persist once quantum effects included?

Including Quantum Effects

Perturbative structure very constrained due to first-order propagator,

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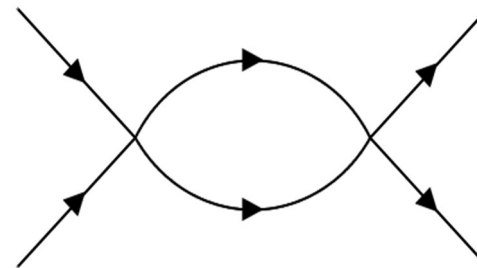
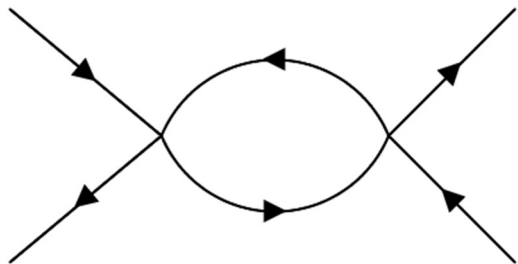
$$G(t, p) = \frac{\Theta(t)}{2v} \exp\left(\frac{ip^2}{2v}\right)$$

In particular, closed particle loops vanish:

$$\text{closed loop} \sim \prod_i \Theta(t_{i+1} - t_i) = 0$$

$t_{N+1} = t_1$ ←

Even with loop energy insertions, can show that these diagrams can only contribute power-law divergences (which cancel via SUSY)



Including Quantum Effects

Two-point and three-point functions completely protected from log-divergences: however, for four-point functions find divergences, e.g.

$$\Gamma_{1-loop}^{(4)} = - \int \frac{d^3 p_i}{(2\pi)^3} \omega_1^A \omega_2^B \bar{\omega}_3^C \bar{\omega}_4^D V_{ABCD}(p_i)$$
$$V_{ABCD} = (\delta_{AB}\delta_{CD} - \delta_{AC}\delta_{BD}) \int \frac{d^2 \tilde{p}}{(2\pi)^2} \frac{2}{v\tilde{p}^2} + \text{finite}$$

No divergences found in correlation functions involving y^A : need to add new terms in renormalisation! Obvious choices

$$[X, Y^A][X, Y^B][Y^A, Y^B] , ([X, Y^A][X, Y^B])^2$$

Must be able to do this while preserving SUSY: extremely constraining, seems to pick out a unique candidate one-parameter family of deformations

SUSY preserving? Conformal?

Conclusion and Future Directions

To sum up:

- For each BPS brane in string theory, we have an associated near-BPS limit
- Constructed the M5-brane limit of eleven-dimensional supergravity: saw that it possessed an emergent local scaling symmetry and Poisson equation, and showed how to construct brane solutions
- Took near-BPS limits of D-brane holographic pairs: on QFT side found dynamics reduced to motion on relevant soliton moduli space, with same symmetries as dual non-Lorentzian supergravity solution
- Started including quantum corrections using the example of supersymmetric Galilean Yang-Mills

Conclusion and Future Directions

What next?

GYM + NRST:

- Do we have a SUSY-preserving deformation that leaves the scalar four-point function UV-finite at one loop? Does this extend to the whole theory?
- What does the theory look like at the quantum level for a general gauge group? In particular, what happens at large- N ?
- Can we construct the worldsheet dual in NRST and match observables on both sides?

Dp-Brane Limits:

- Can we construct full quantum dynamics from an intrinsically bulk perspective for arbitrary asymptotics?
- How does relativistic bulk structure emerge at large- N ?

Thanks for
listening!