Prequantization of shifted symplectic structures and transgression

Milena Weiershausen

Mathematical Institute, Georg-August-University Göttingen

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Introduction: Geometric (pre-)quantization

Quantization

Input: Classical system (symplectic manifold of classical states with observables given by smooth functions on it)
Output: Quantum system (Hilbert space of quantum states with observables given by continuous operators acting on it).
Dirac's axioms: (cited based on Carosso 2022)

1. Linearity: $Q(\lambda a + \mu b) = \lambda Q(a) + \mu Q(b)$
2. Normalization: $Q(1) = id$
3. Quantum condition: $Q(\{a, b\}) = \frac{1}{i\hbar}[Q(a), Q(b)]$
4. Minimality (or Power rule): $Q(a^n) = Q(a)^n$

However, it was shown by Groenewold and van Hove (cited based on Carosso 2022) that a functor satisfying all four of these axioms cannot exist.

Geometric Quantization

Ordinary Setting:

(M, ω) symplectic manifold

$\downarrow$ (prequantization)

$(\pi : L \to M)$ complex line bundle

state space: sections of L

$\mathcal{H}_{\mathrm{pre}} := L^2(M, L)$

$\downarrow$ (polarization)

$\mathcal{H}$ restrict to smaller Hilbert space

Symplectic Manifolds

Definition

A *symplectic manifold* is a smooth manifold M equipped with a closed non-degenerate differential 2-form ω (*symplectic form*).

Example (Single particle in $\mathbb{R}^3$)

States: phase space $T^*\mathbb{R}^3 \cong \mathbb{R}^6$, **Energy:** Hamiltonian $H(q_i, p_i)$.
Standard symplectic form $\omega = \sum_{i=1}^{3} dp_i \wedge dq_i$.
Hamiltonian vector field $i_{X_H}\omega = -dH$ has the form

$$X_H = \sum_{i=1}^{3} \left(\frac{\partial H}{\partial p_i}\frac{\partial}{\partial q_i} - \frac{\partial H}{\partial q_i}\frac{\partial}{\partial p_i} \right).$$

Dynamics determined by $\frac{df}{dt} = X_H(f)$.

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}.$$

Prequantum Line Bundles

$H^2(M, \mathbb{Z})$ characterizes Hermitian line bundles over M with connection.

Theorem (Chern-Weil)

Let $\pi : L \to M$ be a complex line bundle with connection ∇. Then the first Chern class $c_1(L, \nabla) = [\frac{1}{2\pi i}\mathrm{curv}(L, \nabla)] \in H^2(M, \mathbb{Z})$ is a topological invariant of L.

If $[\omega] \in H^2(M, \mathbb{R})$ is integral, i.e. ω defines an integral cohomology class, then we can find a corresponding line bundle $L \to M$ with connection ∇ such that $[\mathrm{curv}(L, \nabla)] \sim c_1(L, \nabla) \sim [\omega]$.
This is called a *prequantum line bundle*.

Geometric Prequantization

Definition (cf. Kostant 1970 and Souriau 1997)

For a prequantum line bundle (L, ∇) of (M, ω), the space $L^2(M, L)$ equipped with the inner product

$$\langle s_1, s_2 \rangle := \frac{1}{(2\pi)^n} \int_M h(s_1, s_2) \frac{\omega^n}{n!}$$

where $h(s_1, s_2)(p)$ denotes the Hermitian inner product on L_p, is a Hilbert space called the *prequantum Hilbert space* of (M, ω) and denoted $\mathcal{H}_{\text{pre}}$.

Proposition (cf. Kostant 1970 and Souriau 1997)

For $f \in \mathcal{C}^\infty(M)$, define $Q(f) := -i\nabla_{X_f} + m_f$, where X_f is the Hamiltonian vector field and m_f the operator of multiplication by f. The assignment $f \mapsto Q(f)$ is a prequantization.

Why do we care about higher structures?

Higher Structures

- Quantization of Poisson structures,
- Quotient type singularities,
- TFTs.

"Besides mechanics, quantization is also relevant for field theories. [...] In topological field theories [...] the phase space of an n-dimensional classical field theory associated to a closed d-dimensional manifold carries an $(n-d-1)$-shifted symplectic structure." (Safronov 2020)

What are higher Lie group(oid)s?

Lie Groupoids

Definition

A *Lie groupoid* $G_1 \rightrightarrows G_0$ consists of

- a manifold of objects G_0,
- a manifold of arrows G_1,
- surjective submersions $s, t : G_1 \twoheadrightarrow G_0$,
- smooth structure maps

$$u : G_0 \longrightarrow G_1,\ x \mapsto id_x,$$
$$i : G_1 \longrightarrow G_1,\ g \mapsto g^{-1},$$
$$m : G_1 \times_{s,G_0,t} G_1,\ (g,h) \mapsto g \circ h,$$

forming category with only invertible arrows.

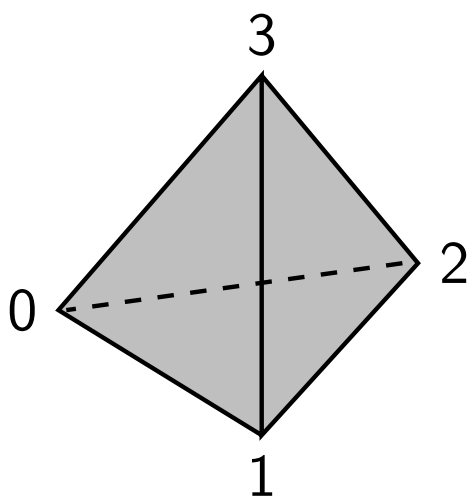
Lie n-Groupoids

Definition

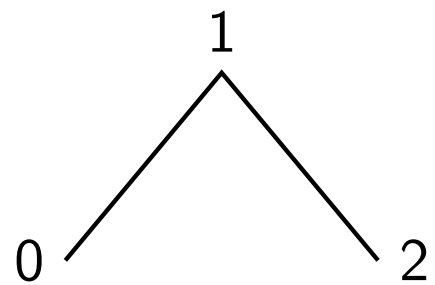
A *simplicial manifold* is a functor $X_\bullet : \Delta^{op} \to \mathrm{Mfd}$.

- A tower of manifolds X_m
- smooth face maps $d_i^m : X_m \to X_{m-1}$ for $i = 0, \ldots, m$
- smooth degeneracy maps $s_i^m : X_m \to X_{m+1}$ for $i = 0, \ldots, m-1$
- simplicial identities.

$\Delta[m]$:



$\Lambda[m, j]$:



Lie n-Groupoids

Definition

A *Lie n-groupoid* $X_\bullet$ is a simplicial manifold that satisfies:

1. Kan(m, j) for all $m \geq 1$ and $0 \leq j \leq m$.
2. Kan!(m, j) for all $m \geq n+1$ and $0 \leq j \leq m$.

where

- Kan(m, j) means the restriction map $\mathrm{Hom}(\Delta[m], X_\bullet) \to \mathrm{Hom}(\Lambda[m, j], X_\bullet)$ is a surjective submersion.
- Kan!(m, j) means the restriction map $\mathrm{Hom}(\Delta[m], X_\bullet) \to \mathrm{Hom}(\Lambda[m, j], X_\bullet)$ is an isomorphism.

A Lie n-group is a Lie n-groupoid $X_\bullet$ where $X_0 = pt$.

Lie n-Groupoids

Example

- Any manifold M can be viewed as the unit Lie groupoid $M \rightrightarrows M$ with constant simplicial nerve. This is denoted $\underline{M}$.
- A Lie group G can be viewed as a Lie groupoid $G \rightrightarrows pt$ with the simplicial nerve $\ldots G \times G \equiv\!\!\!\equiv G \rightrightarrows pt$. This is denoted BG.
- A Lie group action $G \times M \to M$ gives the action groupoid $G \times M \rightrightarrows M$ with nerve $\ldots G \times G \times M \equiv\!\!\!\equiv G \times M \rightrightarrows M$.
  In particular, for G acting on itself by conjugation, we get the AMM groupoid denoted $[G/G]$ (Alekseev, Malkin, and Meinrenken 1998).
- Crossed modules are Lie 2-groups. The string group $\mathrm{String}(G)$ can be described as a Lie 2-group central extension $BU(1) \to \mathrm{String}(G) \to \underline{G}$.

Hypercovers

$$\partial_m(X_\bullet) := \mathrm{Hom}(\partial\Delta[m], X_\bullet)$$

Definition

$f : X_\bullet \to Y_\bullet$ is a *hypercover* if the maps

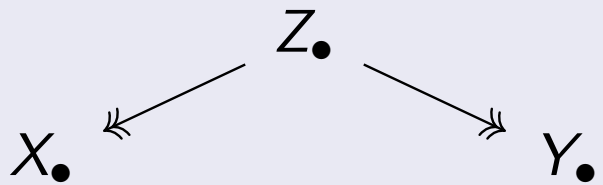
$$q_i := ((d_0, \ldots, d_m), f_m) : X_m \to \partial_m(X_\bullet) \times_{\partial_m(Y_\bullet)} Y_m$$

are a surj subm for $0 \leq m \leq n-1$ and an isom for $m = n$.

In particular, $q_0 = f_0$ is a surjective submersion.

Definition

$X_\bullet$ and $Y_\bullet$ are ME if there exists a $Z_\bullet$ and a zig-zag of hypercovers



Hypercovers

Example

- The pair groupoid $M \times M \rightrightarrows M$ is Morita equivalent to the unit groupoid $pt \rightrightarrows pt$ via the hypercover to the point.
- BG is Morita equivalent to the infinite dimensional model

$$BG_{\mathrm{inf}} := \ldots \Omega G \threerightarrows P_e G \rightrightarrows pt$$

via the hypercover $E : BG_{\mathrm{inf}} \twoheadrightarrow BG$ with $E_1(\gamma) = \gamma(1)$ and $E_2(\tau) = (\tau(\frac{2}{3})\tau(\frac{1}{3})^{-1}, \tau(\frac{1}{3}))$ (cf. Cueca and Zhu 2023).

- $[G/G]$ is Morita equivalent to the infinite dimensional model

$$\Gamma := \ldots LG \times LG \times L\mathfrak{g} \threerightarrows LG \times L\mathfrak{g} \rightrightarrows L\mathfrak{g},$$

via $F_k(\lambda_1, \ldots, \lambda_k, A) = (\mathrm{Hol}(A), \lambda_1(0), \ldots, \lambda_k(0))$ under $L\mathfrak{g} \cong L\mathfrak{g}^* \cong \Omega^1(\mathbb{S}^1, \mathfrak{g})$ (cf. Behrend, Xu, and Zhang 2003).

What are higher symplectic structures?

Recall: Symplectic Manifolds

Definition

A *symplectic manifold* is a smooth manifold M equipped with a **closed non-degenerate differential 2-form** ω (*symplectic form*).

For $X \in \mathfrak{X}(M)$, we get an *associated* 1*-form* defined as

$$\omega_X : \mathfrak{X}(M) \to \mathcal{C}^\infty(M), Y \mapsto \omega_X(Y) := \omega(X, Y).$$

This gives us an **isomorphism**

$$\begin{aligned} \omega^\flat : \quad TM &\xrightarrow{\cong} T^*M \\ (v \in T_pM) &\mapsto \iota_v \omega_p \end{aligned}$$

m-Shifted k-Forms

$$\begin{array}{ccccccc}
\vdots & & \vdots & & \vdots & & \\
\uparrow d & & \uparrow d & & \uparrow d & & \\
\Omega^2(X_0) & \xrightarrow{\delta} & \Omega^2(X_1) & \xrightarrow{\delta} & \Omega^2(X_2) & \xrightarrow{\delta} & \dots \\
\uparrow d & & \uparrow d & & \uparrow d & & \\
\Omega^1(X_0) & \xrightarrow{\delta} & \Omega^1(X_1) & \xrightarrow{\delta} & \Omega^1(X_2) & \xrightarrow{\delta} & \dots \\
\uparrow d & & \uparrow d & & \uparrow d & & \\
\Omega^0(X_0) & \xrightarrow{\delta} & \Omega^0(X_1) & \xrightarrow{\delta} & \Omega^0(X_2) & \xrightarrow{\delta} & \dots
\end{array}$$

de Rham differential $d : \Omega^q(X_p) \to \Omega^{q+1}(X_p)$
simplicial differential $\delta : \Omega^q(X_{p-1}) \to \Omega^q(X_p)$, $\delta = \sum_{i=0}^{p}(-1)^i d_i^*$.

m-Shifted k-Forms

$$\begin{array}{ccccccc}
\vdots & & \vdots & & \vdots & & \\
\uparrow d & & \uparrow d & & \uparrow d & & \\
\Omega^2(X_0) & \xrightarrow{\delta} & \Omega^2(X_1) & \xrightarrow{\delta} & \Omega^2(X_2) & \xrightarrow{\delta} & \dots \\
\uparrow d & & \uparrow d & & \uparrow d & & \\
\Omega^1(X_0) & \xrightarrow{\delta} & \Omega^1(X_1) & \xrightarrow{\delta} & \Omega^1(X_2) & \xrightarrow{\delta} & \dots \\
\uparrow d & & \uparrow d & & \uparrow d & & \\
\Omega^0(X_0) & \xrightarrow{\delta} & \Omega^0(X_1) & \xrightarrow{\delta} & \Omega^0(X_2) & \xrightarrow{\delta} & \dots
\end{array}$$

The total complex $(K^\bullet, D)$:
$K^n := \bigoplus_{p+q=n} \Omega^q(X_p)$, $D = \delta + (-1)^p d$

m-Shifted Presymplectic k-Forms

Definition (introduced by Getzler (2014), cf. Cueca and Zhu (2023))

An *m-shifted k-form* $\alpha_\bullet$ on $X_\bullet$ is of the form

$$\alpha_\bullet = \sum_{i=0}^{m} \alpha_i \text{ with } \alpha_i \in \hat{\Omega}^{k+m-i}(X_i).$$

$\alpha_\bullet$ is *closed* if $D\alpha_\bullet = 0$.
A closed m-shifted 2-form is called an *m-shifted presymplectic form*.

We can alternatively write the filtration as

$$F^k\widehat{\Omega}^q(X_p) := \begin{cases} 0, & \text{if } q < k, \\ \widehat{\Omega}^q(X_p), & \text{if } q \geq k. \end{cases}$$

An m-shifted k-form is an element $\alpha_\bullet \in F^k\widehat{\Omega}^{k+m}$.

IM-Forms and Nondegeneracy

Definition (introduced by Getzler (2014), cf. Cueca and Zhu (2023))

For $l \in \mathbb{Z}$, $v \in (\mathcal{T}_l K)_x \subseteq T_x K_l$, and $w \in (\mathcal{T}_{m-l} K)_x \subseteq T_x K_{m-l}$ in the tangent complex at $x \in K_0$, we define the IM-form

$$\lambda_x^{\omega\bullet}(v, w) \coloneqq \sum_{\sigma \in \mathrm{Shuff}_{l,m-l}} (-1)^\sigma \omega_m(T(s_{\sigma(m-1)} \dots s_{\sigma(l)})v, T(s_{\sigma(l-1)} \dots s_{\sigma(0)})w).$$

An m-shifted 2-form $\omega_\bullet$ is called *non-degenerate* if $\lambda^{\omega\bullet}$ induces a quasi-isomorphism between $\mathcal{T}_\bullet K$ and $\mathcal{T}_\bullet^*[m]K$.

Definition (introduced by Getzler (2014), cf. Cueca and Zhu (2023))

An *m-shifted symplectic Lie n-groupoid* is a Lie n-groupoid $K_\bullet$ with a closed, normalised, and non-degenerate m-shifted 2-form $\omega_\bullet$.

Shifted Symplectic Lie n-Groupoids

Assume G to be compact, simple, simply connected.

Example

- $(BG, \frac{1}{2}(\Omega - \eta))$ is a 2-shifted symplectic Lie 1-group (Cueca and Zhu 2023):

  $\Omega = \langle d_2^* \theta^l, d_0^* \theta^r \rangle \in \Omega^2(G \times G)$ (Brylinski-Weinstein 2-form),

  $\eta = \frac{1}{6} \langle \theta^l, [\theta^l, \theta^l] \rangle \in \Omega^3(G)$ (Cartan 3-form).

- $(BG_{\mathrm{inf}}, \omega_{Seg})$ is a 2-shifted symplectic Lie 2-group (Cueca and Zhu 2023):

  $$(\omega_{Seg})_\tau(a, b) = \int_{\mathbb{S}^1} \langle \frac{d}{dt} TL_{\tau(t)^{-1}} a(t), TL_{\tau(t)^{-1}} b(t) \rangle dt$$

  for $\tau \in \Omega G$, $a, b \in T_\tau \Omega G$, and L the left translation map.

Shifted Symplectic Lie n-Groupoids

Example

- $([G/G], \omega_{\mathrm{AMM}} + \eta)$ is a 1-shifted symplectic Lie groupoid (Alekseev, Malkin, and Meinrenken 1998):

$$[G/G] = \ldots G \times G \times G \equiv\!\!\!\rightrightarrows G \times G \rightrightarrows G, \quad \Omega_\bullet = \omega_{\mathrm{AMM}} + \eta,$$

$$\omega_{\mathrm{AMM}} = \frac{1}{2}(\langle \mathrm{Ad}\theta^r, \theta^r\rangle - \langle \theta^l + \theta^r, \theta^r\rangle) \in \Omega^2(G \times G),$$

$$\eta = \frac{1}{12}\langle \theta^l, [\theta^l, \theta^l]\rangle \in \Omega^3(G) \text{ (Cartan 3-form)}.$$

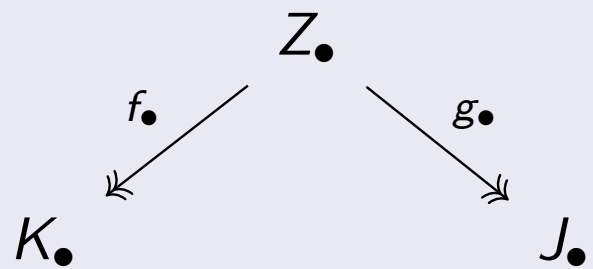
- $(\Gamma, \omega_{LG \times L\mathfrak{g}})$ is a 1-shifted symplectic Lie groupoid (Behrend, Xu, and Zhang 2003): $\omega_{LG \times L\mathfrak{g}} = F^*(\omega_{\mathrm{AMM}} + \eta) + \delta\mu$

$$\mu = \frac{1}{2}\int_0^1 \langle \mathrm{Hol}_s^* \theta^r, \frac{\partial}{\partial s}\mathrm{Hol}_s^* \theta^r\rangle.$$

Symplectic Morita Equivalence

Definition (introduced by Getzler (2014), cf. Cueca and Zhu (2023))

Two m-shifted symplectic Lie n-groupoids $(K_\bullet, \alpha_\bullet)$ and $(J_\bullet, \beta_\bullet)$ are *symplectic Morita equivalent* if there exists another Lie n-groupoid $Z_\bullet$ with an $(m-1)$-shifted 2-form $\Phi_\bullet$ and hypercovers

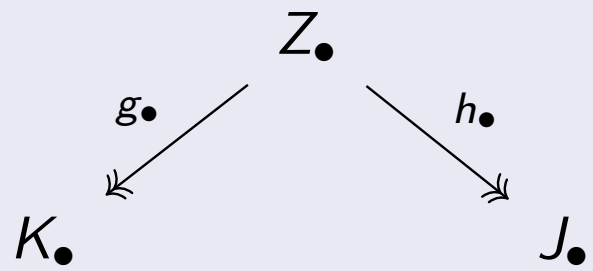


satisfying $f_\bullet^* \alpha_\bullet - g_\bullet^* \beta_\bullet = D\Phi_\bullet$.

Symplectic Morita Equivalence

Proposition (W.)

Let



be a Morita equivalence of Lie n-groupoids and $\alpha_\bullet$ an m-shifted symplectic form on $K_\bullet$.
Then, there is also an induced m-shifted symplectic form $\beta_\bullet$ on $J_\bullet$ and an $(m-1)$-shifted 2-form on $Z_\bullet$ such that the zig-zag of hypercovers becomes a symplectic Morita equivalence.

Symplectic Morita Equivalence

Example

- $(BG, \frac{1}{2}\Omega)$ is symplectic Morita equivalent to $(BG_{\mathrm{inf}}, \omega_{Seg})$ via

$$\begin{array}{ccc} & (BG_{\mathrm{inf}}, \omega_{\bullet}^{P}) & \\ {\scriptstyle id}\swarrow & & \searrow{\scriptstyle E} \\ (BG_{\mathrm{inf}}, \omega_{Seg}) & & (BG, \frac{1}{2}(\Omega - \eta)) \end{array}$$

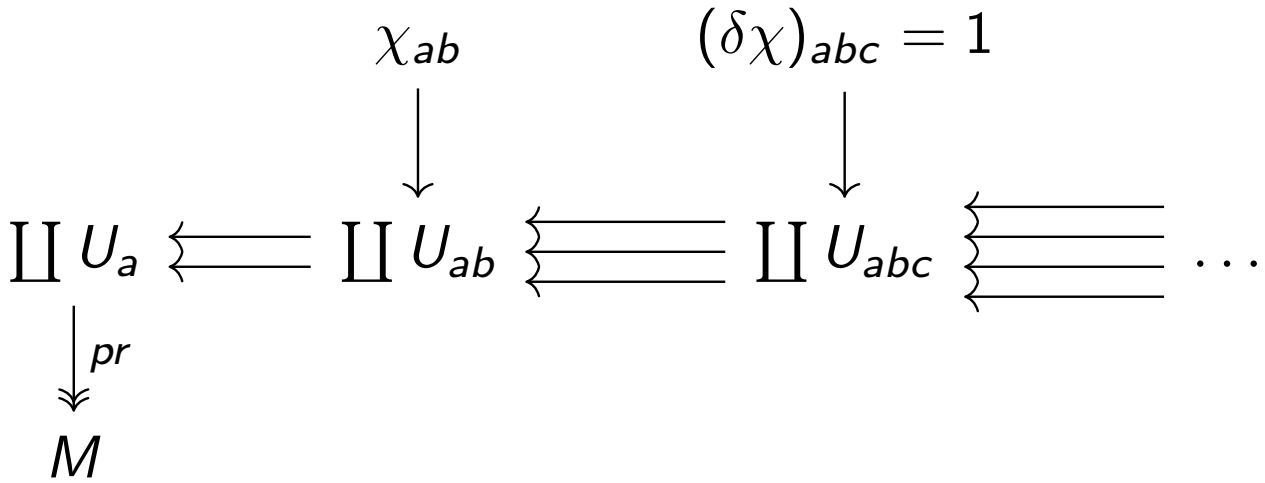
  where $\omega_{\bullet}^{P}$ is a 1-shifted 2-form (Cueca and Zhu 2023).
- $([G/G], \omega_{\mathrm{AMM}} + \eta)$ is symplectic Morita equivalent to $(\Gamma, \omega_{LG \times L\mathfrak{g}})$ via (Behrend, Xu, and Zhang 2003)

$$\begin{array}{ccc} & (\Gamma, \mu) & \\ {\scriptstyle id}\swarrow & & \searrow{\scriptstyle F} \\ (\Gamma, \omega_{LG \times L\mathfrak{g}}) & & ([G/G], \omega_{\mathrm{AMM}} + \eta). \end{array}$$

What are higher prequantum line bundles?

Recall: Complex Line Bundles with Connection

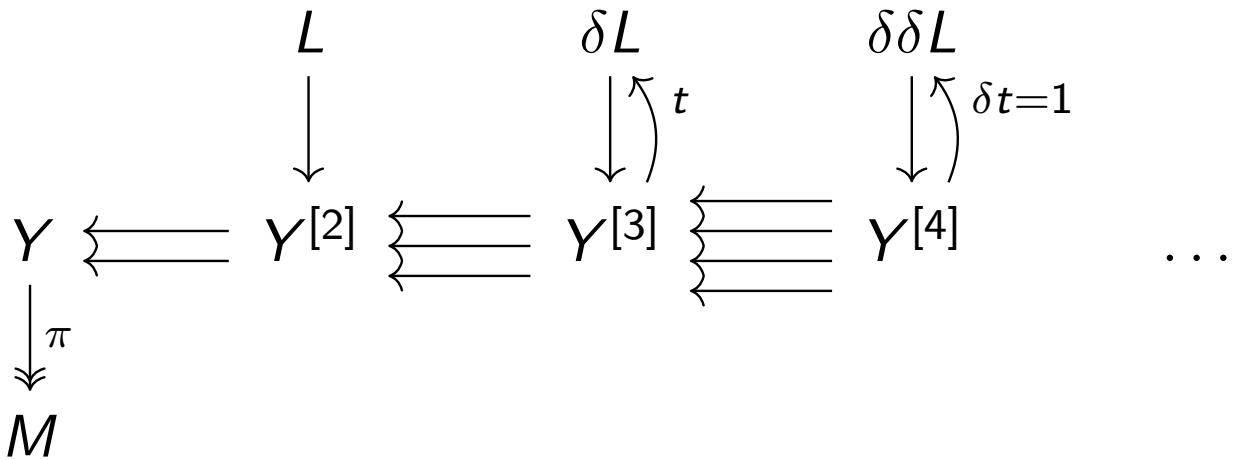
$H^2(M, \mathbb{Z})$ characterizes $\mathbb{C}$-line bundles over M with connection. Locally, a line bundle $L \to M$ can be described by a trivializing cover $\{U_a\}$ and transition functions $\chi_{ab} : U_a \cap U_b \to U(1)$ satisfying $(\delta\chi)_{abc} = \chi_{bc}\chi_{ac}^{-1}\chi_{ab} = 1$ (cf. Meinrenken 2003).



A connection is given locally by connection 1-forms $A_i \in \Omega^1(U_i, i\mathbb{R})$.
The dA_i then glue to a global 2-form $\mathrm{curv}(L) \in \Omega^2(M, i\mathbb{R})$.

Gerbes with Connection

$H^3(M, \mathbb{Z})$ characterizes gerbes over M with connection.



with $Y^{[2]} := Y \times_M Y$.

The connection data is given by a connection 1-form $A \in \Omega^1(Y^{[2]}, i\mathbb{R})$ on L and a 2-form $B \in \Omega^2(Y, i\mathbb{R})$ such that $\mathrm{curv}(L) := dA = -\delta B$.

Gerbes with Connection

Definition (cf. Bunk 2021)

A bundle gerbe $\mathcal{L}$ over M consists of

- a surjective submersion $\pi : Y \twoheadrightarrow M$,
- a line bundle $L \to Y^{[2]}$,
- an isom of line bundles $\mu : pr_{23}^* L \otimes pr_{12}^* L \cong pr_{13}^* L$ over $Y^{[3]}$,
- st. μ satisfies an associativity condition over $Y^{[4]}$:

$$\begin{array}{ccc} pr_{34}^* L \otimes pr_{23}^* L \otimes pr_{12}^* L & \xrightarrow{id\otimes\mu} & pr_{34}^* L \otimes pr_{13}^* L \\ \downarrow{\scriptstyle \mu\otimes id} & & \downarrow{\scriptstyle \mu} \\ pr_{24}^* L \otimes pr_{12}^* L & \xrightarrow{\mu} & pr_{14}^* L \end{array}$$

A connection on $\mathcal{L}$ consists of a connection 1-form A on L and a 2-form $B \in \Omega^2(Y, i\mathbb{R})$ s.t. $\mathrm{curv}(L) := dA = -\delta B$.
There exists a unique 3-form $H \in \Omega^3(M, i\mathbb{R})$ such that $\pi^* H = dB$.
This 3-form H is called the *curvature 3-form* of $(\mathcal{L}, A, B)$.

Simplicial Gerbes with Connection

Definition (based on Bunk 2021)

Let Γ be a simplicial manifold. A Γ-*equivariant bundle gerbe* (i.e. a bundle gerbe over Γ) consists of

- a gerbe $\mathcal{L}$ over Γ_0,
- an isomorphism $\mathcal{P} : \delta\mathcal{L} \cong \mathcal{I}_1$ of gerbes over Γ_1,
- an isomorphism $\mu : \delta\mathcal{P} \cong \mathcal{I}_0$ of line bundles over Γ_2,
- satisfying a coherence condition $\delta\mu = 1$ over Γ_3.

A connection on $(\mathcal{L}, \mathcal{P}, \mu)$ consists of a connection on $\mathcal{L}$ with $\mathrm{curv}(\mathcal{L}) = H$ and a 2-form $B \in \Omega^2(\Gamma_1, i\mathbb{R})$ such that $\mathrm{curv}(\delta\mathcal{L}) = \delta H = dB$ and μ is connection-preserving, i.e. $\delta B = 0$.

Note that in particular, for $\Gamma = \underline{M}$, this is again a gerbe over M. Additionally, note that the curvature (H, B) of this simplicial gerbe is a 1-shifted presymplectic form.

Simplicial 2-Gerbes with Connection

Remark (based on Bunk 2021)

A simplicial 2-gerbe over BG consists of

- a 2-gerbe $\mathcal{T}$ over pt,
- an isom $\mathcal{L} : d_0^*\mathcal{T} \cong d_1^*\mathcal{T}$ of 2-gerbes over G,
- an isom $\mathcal{P} : d_0^*\mathcal{L} \otimes d_2^*\mathcal{L} \cong d_1^*\mathcal{L}$ of gerbes over G^2,
- an isom $\mu : d_0^*\mathcal{P} \otimes d_2^*\mathcal{P} \cong d_1^*\mathcal{P} \otimes d_3^*\mathcal{P}$ of line bundles over G^3,
- satisfying a coherence condition over G^4.

Any 2-gerbe over pt is trivializable (choose trivialization $\mathcal{T} \cong \mathcal{I}_2$). $\mathcal{L}$ is a gerbe over G and the rest of the data says it is a so-called multiplicative bundle gerbe. A connection on this is a 2-shifted presymplectic form and the 4-form on $BG_0 = pt$ is 0.

Shifted Symplectic Prequantization

Definition (based on Bunk 2021)

Let $(\Gamma, \omega_\bullet)$ be an m-shifted symplectic Lie n groupoid. A *prequantization* of $(\Gamma, \omega_\bullet)$ consists of a simplicial m-gerbe $\mathcal{G}$ with connection over Γ such that $\mathrm{curv}(\mathcal{G})_\bullet = \omega_\bullet$.

Example

- $(BG_{\mathrm{inf}}, \omega_{Seg})$ defines generator in $H^4(BG, \mathbb{Z}) \cong \mathbb{Z}$ and corresponds to a prequantization modeled by

$$B\mathrm{String}(G) = \ldots \widetilde{\Omega G} \rightrightarrows P_e G \rightrightarrows pt.$$

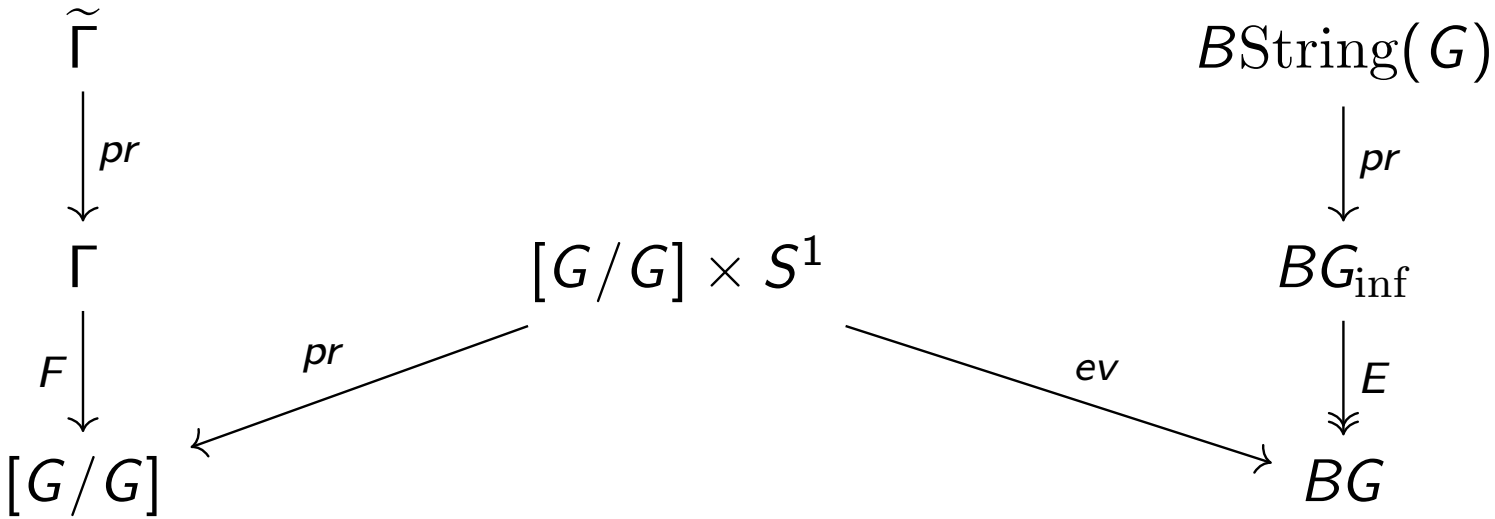
- $(\Gamma, \omega_{LG \times L\mathfrak{g}})$ (under the same conditions as for BG) has

$$\widetilde{\Gamma} = \ldots \widetilde{LG} \times \widetilde{LG} \times L\mathfrak{g} \rightrightarrows \widetilde{LG} \times L\mathfrak{g} \rightrightarrows L\mathfrak{g}$$

(cf. Behrend, Xu, and Zhang 2003).

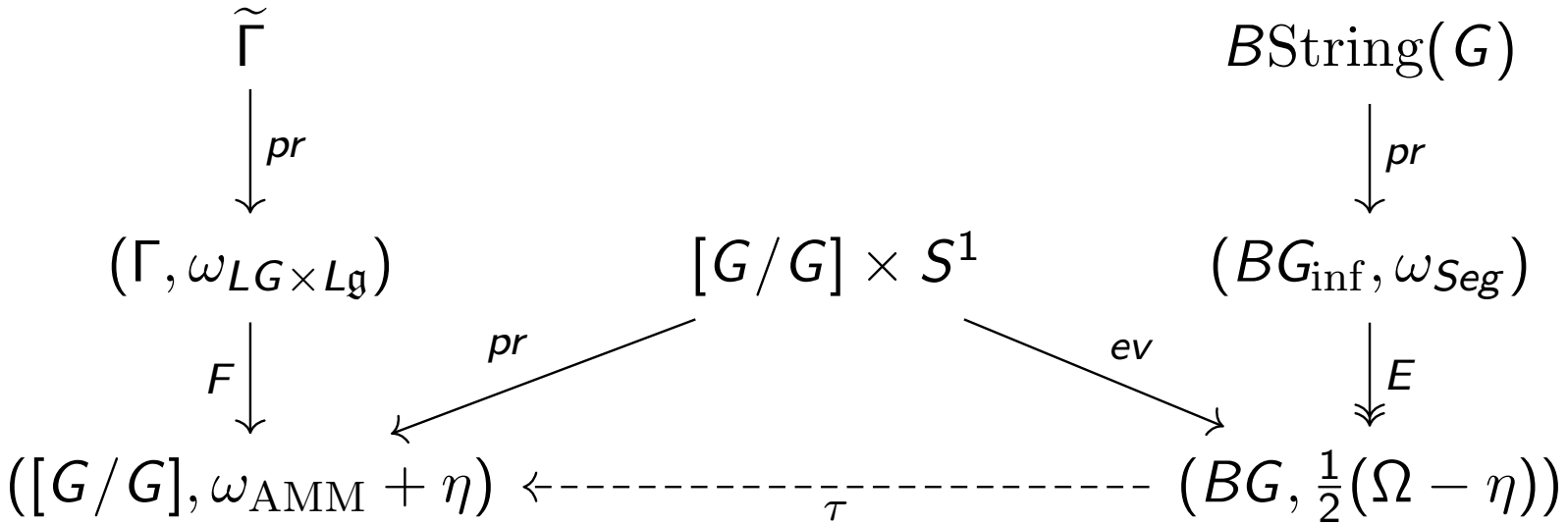
How do these puzzle pieces fit together?

Transgression Picture



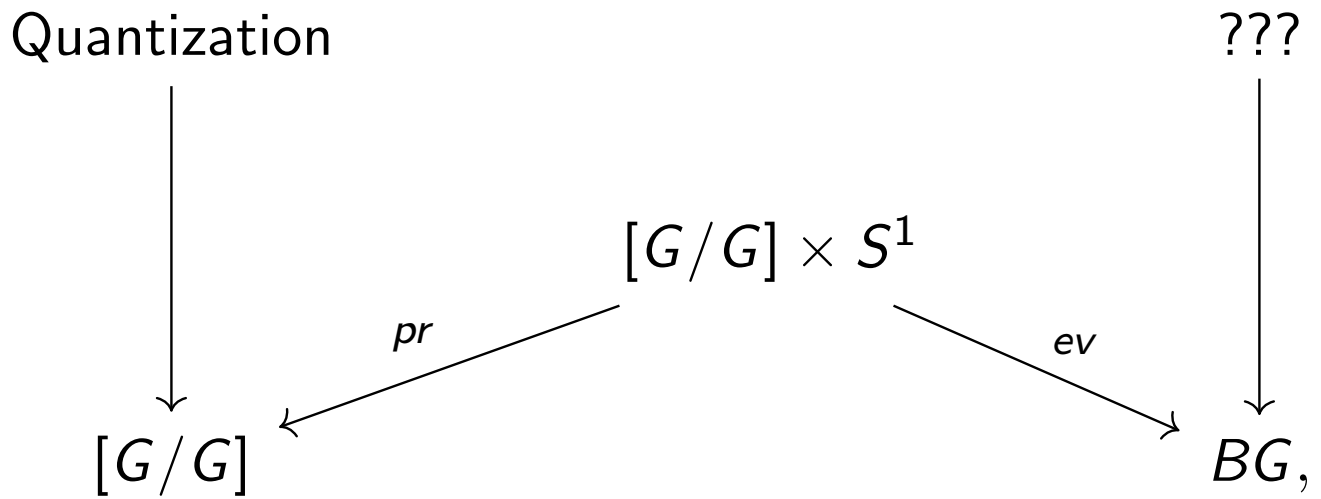
Based on (Safronov 2016).

Transgression Picture



Based on (Safronov 2016).

Outlook



Goal: Quantization of $[G/G]$ (Meinrenken 2010) could inspire quantization of BG.

Thank you!

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